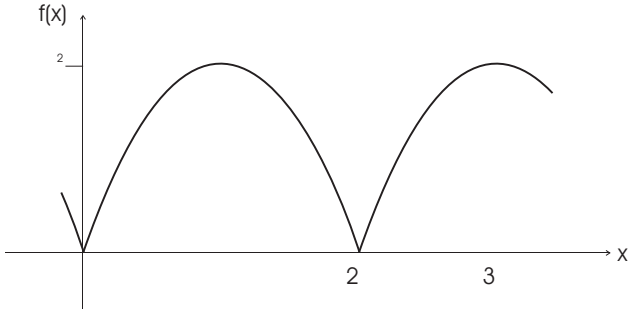
3.11.4 Example: Periodic parabolic function

$f(x) = x(2\pi - x), \;\; 0 \le x \le 2\pi$ periodic continuation



$$\begin{aligned}
\frac{a_0}{2} &= \frac{1}{2\pi}\int_0^{2\pi}(2\pi x - x^2)dx = \frac{1}{2\pi}\left[\pi x^2 - \frac{1}{3}x^3\right]_0^{2\pi} = \frac{2}{3}\pi^2 \\
a_k &= \frac{1}{\pi}\int_0^{2\pi} 2\pi x\cos kx\; dx - \frac{1}{\pi}\int_0^{2\pi} x^2\cos kx\; dx \\
&= \frac{1}{\pi}\left[2\pi\left(\frac{\cos kx}{k^2} + \frac{x\sin kx}{k}\right) - \left(\frac{2x}{k^2}\cos kx + \left(\frac{x^2}{k} - \frac{2}{k^3}\right)\sin kx\right)\right]_0^{2\pi} \\
\frac{1}{\pi}\left(-\frac{4\pi}{k^2}\right) &= -\frac{4}{k^2}
\end{aligned}$$

Thus: $f(x) = \frac{2}{3}\pi^2 - 4\sum_{k=1}^{\infty}\frac{\cos kx}{k^2} \quad \Rightarrow \quad$ Due to $1/k^2$ fast converging series