2.7.2 Matrix multiplication commutative?

$$\left.\begin{array}{l}\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix} \\ \\ \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 4 & 4 & 1 \end{pmatrix}\end{array}\right\} \neq$$

$\Rightarrow$ This clearly means that the multiplication of matrices is not commutative, i.e.

$$\tilde{A}\tilde{B} \neq \tilde{B}\tilde{A} \text{ in general!!}$$

But at least:

$$\begin{aligned} \tilde{A}(\tilde{B}\tilde{C}) &= (\tilde{A}\tilde{B})\tilde{C} \qquad \text{associative rule} \\ \text{and } \tilde{A}(\tilde{B}+\tilde{C}) &= \tilde{A}\tilde{B}+\tilde{A}\tilde{C} \qquad \text{distributive rule} \\ \text{but: } (\tilde{A}+\tilde{B})(\tilde{A}-\tilde{B}) &= \tilde{A}^2 - \tilde{A}\tilde{B} + \tilde{B}\tilde{A} - \tilde{B}^2 \neq \tilde{A}^2 - \tilde{B}^2 \text{ in general since } \tilde{A}\tilde{B} \neq \tilde{B}\tilde{A} \end{aligned}$$

Example:

$$\underset{2\times 4}{\begin{pmatrix} 5 & 2 & -2 & 3 \\ 9 & 2 & -3 & 4 \end{pmatrix}} \underset{4\times 3}{\begin{pmatrix} 2 & 2 & 2 \\ -1 & 3 & -5 \\ 16 & 8 & 24 \\ 8 & 0 & 16 \end{pmatrix}} = \underset{2\times 3}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}$$

$\Rightarrow$ Matrices do have zero divisors (see example)!! Thus, if $\tilde{A}\cdot\tilde{B} = \tilde{0}$ and $\tilde{A} \neq \tilde{0}$ then in general not $\tilde{B} = \tilde{0}$, and if $\tilde{A}\tilde{B} = \tilde{A}\tilde{C}, \tilde{A} \neq \tilde{0}$ than not $\tilde{B} = \tilde{C}$!!! (Example in homework)

Note, that this would be trivial for real or complex numbers $\Rightarrow$ Matrices are different from numbers (which makes them relevant for quantum mechanics)