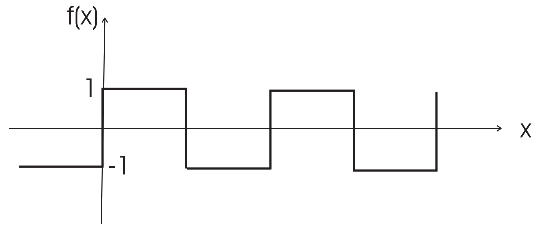
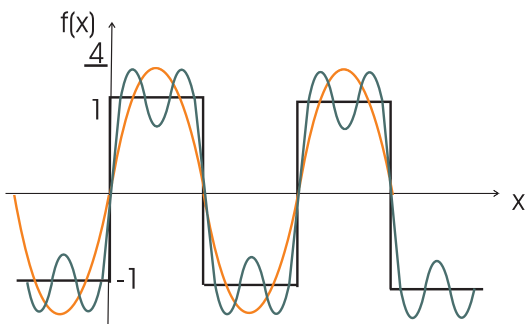
3.11.1 Example: Periodic step function

$$f(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x \leq \pi \\ -1 & \text{if} \quad \pi \leq x \leq 2\pi \end{cases} \quad \text{and periodic continuation}$$



Important note:

$$\begin{aligned}
f(x) &= -f(-x) \rightarrow \text{anti-symmetric} \rightarrow \text{cos-trans are zero!} \int\limits_0^{2\pi} f(x)\cos(kx)dx = 0 \\
b_k &= \frac{1}{\pi}\int\limits_0^{2\pi} f(x)\sin(kx)dx = \frac{1}{\pi}\int\limits_0^{\pi} 1\sin(kx)dx + \frac{1}{\pi}\int\limits_\pi^{2\pi}(-1)\sin(kx)dx \\
&= \frac{1}{\pi}\left(\frac{-\cos(kx)}{k}\Bigg|_0^\pi - \frac{-\cos(kx)}{k}\Bigg|_\pi^{2\pi}\right) = \frac{1}{\pi k}\left(1-(-1)^k+1-(-1)^k\right) \\
&= \frac{2}{\pi k}\left(1-(-1)^k\right) = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{4}{\pi k} & \text{if } k \text{ odd, } k = 2n+1 \end{cases} \\
\text{Thus: } f(x) &= \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)x}{2n+1} = \frac{4}{\pi}\left(\sin x + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \ldots\right)
\end{aligned}$$



$\rightarrow$ approximation by trigonometric functions due to the factor $1/(2n+1)$ the series is slowly converging (best with $n \rightarrow \infty$)