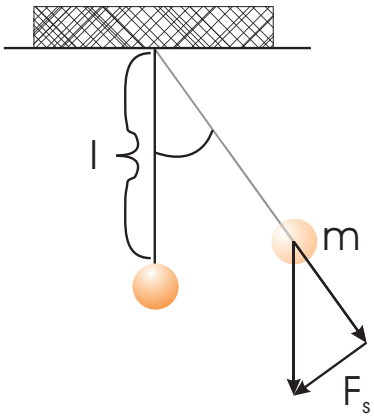
3.6.1 Example: Transformation of a non linear problem into a linear problem

Motion of a pendulum:



$$s = l\varphi \qquad F_s = -mg\sin\varphi$$

Newton:

$$\begin{aligned} F_s = m\ddot{s} \quad &\rightarrow \quad m\ddot{s} = -mg\sin\varphi \\ l\ddot{\varphi} + g\sin\varphi = 0 \quad &\rightarrow \quad \ddot{\varphi} + \omega^2\sin\varphi = 0 \qquad (\star) \\ &\qquad\qquad \varphi(t) = ? \text{ function is looked for ?} \end{aligned}$$

Eq. $(\star)$ is extremely complicated since it is non-linear because of the $\sin\varphi$. For small φ:
$\sin\phi \approx \varphi \quad |\sin\varphi - \varphi| \leq \frac{\varphi^3}{3!} \approx 10^{-3}$ for $\varphi \leq 10°$

$$\rightarrow \ \ddot{f}(t) + \omega^2 f(t) = 0 \quad \frac{d^2\varphi}{dt^2} + \omega^2\varphi(t) = 0 \ \leftarrow \text{ linear equation}$$

<u>See also exercises:</u>

$$f(t) = A_1 e^{i\omega t} + A_2 e^{-i\omega t} = \varphi\cos(\omega t) + \frac{\varphi}{\omega}\sin(\omega t)$$