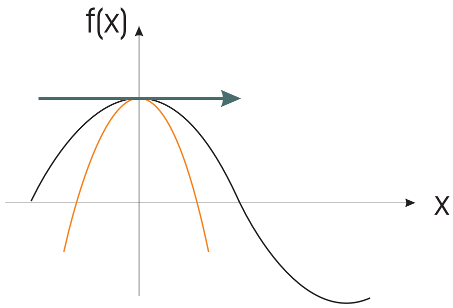
3.5.3 Example: Taylor expansion of cos function



$$
\begin{aligned}
f(x) &= \cos x; \qquad x_0 = 0 \\
f(x) &\approx \cos 0 + x(-\sin 0) \approx 1 \\
f(x) &\approx 1 - \frac{1}{2}x^2 \\
f'(x) &= -\sin x \to f''(x) = -\cos x \to f'''(x) = \sin x \\
& \qquad \to f''''(x) = \cos x = f(x) \\
\Rightarrow f(x) &= \sum_{n=0}^{\infty} \frac{(-1)^b}{(2n)!} x^{2n} = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots
\end{aligned}
$$

$$
\begin{aligned}
f(x) &= e^{ix} \\
&= \sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = \sum_{n=0}^{\infty} \frac{i^n x^n}{n!} = 1 + ix - \frac{1}{2!}x^2 - \frac{i}{3!}x^3 + \frac{1}{4!}x^4 + \dots \\
&= \left(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots\right) + i\left(x - \frac{x^3}{3!} + \dots\right) = \cos x + i \sin x
\end{aligned}
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