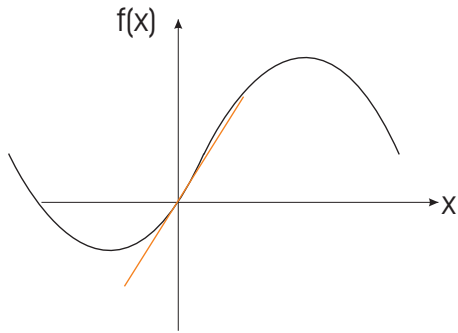### 3.5.2 Example: Taylor expansion of sin function

$$
\begin{aligned}
f(x) &= \sin x \qquad x_0 = 0 \\
f(x) &\approx \sin 0 + x \cos 0 = x \\
f'(x) &= \cos x \to f''(x) = -\sin x \to f'''(x) = -\cos x \\
&\to f''''(x) = \sin x = f(x)
\end{aligned}
$$

$$
\to f^n(0) = \begin{cases}
1 & \text{if} \quad n = 2k+1, \ k \text{ even} \\
-1 & \text{if} \quad n = 2k+1, \ k \text{ odd} \\
0 & \text{if} \quad n = 2k
\end{cases}
$$

$$
\to f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots
$$