4.1 Basic requirements

$$\begin{aligned}
\vec{x} &= \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} \in \mathbb{R}^N \text{ vector and } N \text{ dimensional space} \\
(\vec{x}_k) &= \vec{x}_0, \vec{x}_1, \vec{x}_2, \ldots \quad \text{vector sequence} \\
&= \begin{pmatrix} x_{1,0} \\ x_{2,0} \\ \vdots \\ x_{N,0} \end{pmatrix}, \begin{pmatrix} x_{1,1} \\ x_{2,1} \\ \vdots \\ x_{N,1} \end{pmatrix}, \ldots
\end{aligned}$$

Definition 36 *vector sequence $(\vec{x}_k) \in \mathbb{R}^N$ converges to $\vec{a} \in \mathbb{R}^N$ if*

$$\begin{aligned}
x_{1,k} &\to a_1 \\
x_{2,k} &\to a_2 \\
\vdots & \quad \vdots \\
x_{N,k} &\to a_N
\end{aligned}$$

each single component does converge in the sense of 1D convergence

$$\lim_{k\to\infty} \vec{x}_k = \vec{a} \Leftrightarrow \begin{pmatrix} x_{1,k} \\ \vdots \\ x_{N,k} \end{pmatrix} \to \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix}$$

sum with vector series:

$$\sum_{k=0}^{\infty} \vec{x}_k = \begin{pmatrix} \sum x_{1,k} \\ \sum x_{2,k} \\ \vdots \\ \sum x_{N,k} \end{pmatrix}$$

Distance between points:

$$|\vec{x} - \vec{y}| = \left(\sum_{j=1}^{N} (x_j - y_j)^2 \right)^{\frac{1}{2}}$$

$$\lim_{k\to\infty} \vec{x}_k = \vec{a} \Rightarrow |\vec{x}_k - \vec{a}|^2 = \sum_{j=1}^{N} (x_{j,k} - a_j)^2 \to 0 \quad \textit{if} \quad k \to \infty$$