2.13 Eigenvalues and Eigenvectors

Definition 28 *$\tilde{A}$ $N \times N$ matrix $\vec{x}$ $N \times 1$ vector is called Eigenvector if*

$$\tilde{A}\vec{x} = \lambda\vec{x}$$

where the $\lambda \in \mathbb{R}$ or $\mathbb{C}$ are called Eigenvalues of $\tilde{A}$.

Example

$$\tilde{A} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\tilde{A}\vec{x} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} = -1 \cdot \vec{x} \Rightarrow \begin{array}{ll} \vec{x} & \text{Eigenvector} \\ \lambda = -1 & \text{Eigenvalue} \end{array}$$

General treatment

$$\tilde{A}\vec{x} = \lambda\vec{x} \quad \Leftrightarrow \quad \underbrace{\left(\tilde{A} - \lambda\tilde{I}\right)}_{\text{homogeneous system for } \vec{x}} \quad \vec{x} = \vec{0}$$

$$\Rightarrow \text{ only soluble if } \det(\tilde{A} - \lambda\tilde{I}) = 0$$

Thus:

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} - \lambda & & a_{2N} \\ \vdots & & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} - \lambda \end{vmatrix} = 0$$

Def.: $P(\lambda)$ is the characteristic polynomial associated with $\tilde{A}$

$$P(\lambda) = (-1)^N \lambda^N + \alpha_{N-1}\lambda^{N-1} + \ldots + \alpha_1\lambda + \alpha_0 = 0$$

characteristic polynomial!!
Polynomial equation for λ