2.10 Regular matrix, inverse matrix

Definition 26 *if* $\det(\tilde{A}) \neq 0$ *the* $\tilde{A}$ *is called a* <u>*regular matrix*</u>*, otherwise irregular or singular*

<u>inverse matrix:</u>

Definition 27 $\tilde{A}$ $N \times N$ *Matrix with* $\det(\tilde{A}) \neq 0$ *then* $\tilde{A}^{-1}$ *defined by* $\tilde{A}\tilde{A}^{-1} = \tilde{A}^{-1}\tilde{A} = \tilde{I}$*.* $\tilde{A}^{-1}$ *is called the inverse matrix with respect to* $\tilde{A}$*.* $\tilde{A}^{-1}$ *is given by the following formula:*

$$\tilde{A}^{-1} = \frac{1}{\det(\tilde{A})} \begin{pmatrix} \backslash & & / \\ & A_{jk} & \\ / & & \backslash \end{pmatrix}^{\top}_{k,j=1,\ldots,N},$$

where A_{jk} *are the cofactors of* a_{jk} *in* $\tilde{A}$*.*

<u>Note:</u> We still do not define a matrix division since

$(\tilde{A}, \tilde{B}, \tilde{X}_1, \tilde{X}_2$ $N \times N$ matrices, $\det(\tilde{B}) \neq 0)$
$\tilde{B}\tilde{X}_1 = \tilde{A} \;\;\Rightarrow\;\; \tilde{X}_1 = \tilde{B}^{-1}\tilde{A}$
$\tilde{X}_2\tilde{B} = \tilde{A} \;\;\Rightarrow\;\; \tilde{X}_2 = \tilde{A}\tilde{B}^{-1} \neq \tilde{B}^{-1}\tilde{A}$ in general
$\rightarrow$ thus, division cannot be properly defined!! (Scalars: $b^{-1}a = ab^{-1} = \frac{a}{b}$)