1.1 Recapitulation of basic requirements

- Basic elementary functions as (section 2.3):
  e^x, $\ln x$, a^x, $\log_a x$, x^n, $\sum_{k=0}^{K} a_k x^k$
  $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\arcsin x$, $\arccos x$, $\arctan x$, $\operatorname{arccot} x$

- "Nearly" elementary functions:
  $\sinh x$, $\cosh x$, $\tanh x$, $\coth x$, $\operatorname{arsinh} x$, $\operatorname{arcosh} x$, $\operatorname{artanh} x$, $\operatorname{arcoth} x$

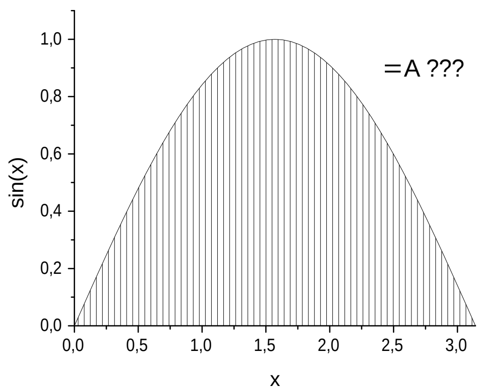
- Properties of these functions, e.g. $e^x e^y = e^{x+y}$
  $\sin(x+y) = \sin x \cos y + \cos x \sin y$

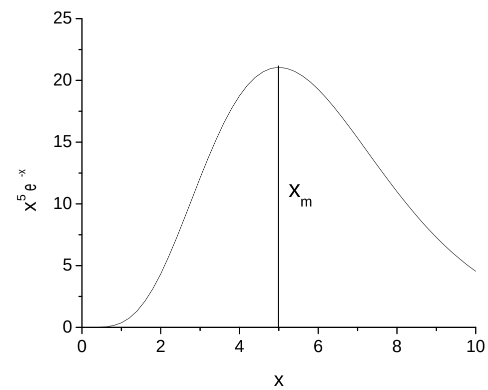
- Calculus of functions with one variable will be briefly recapitulated (see section 3.1), e.g.

  I. $\frac{d}{dx}(x^n) = nx^{n-1}$, $f(x) = x^n$ $f'(x) = nx^{n-1}$

  II. $\int e^{-x} = -e^{-x} + C$

  III. $A = \int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = -(-1) - (-1) = 2$ (units)



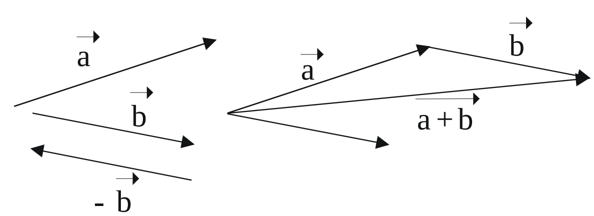


  IV. Maximum of $f(x) = x^5 e^{-x}$ for $x > 0$:
  $f'(x) = 5x^4 e^{-x} - x^5 e^{-x} = e^{-x} x^4 (5 - x)$, $f'(x_m) = 0 \Rightarrow x_m = 5$

- Vector algebra in 3D (see section 2.6)

  I. Adding vectors

  $\vec{a} + \vec{b}$
  $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$
  $\vec{0} \equiv$ Zero Vector

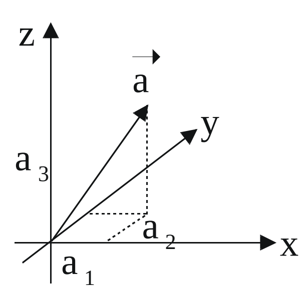


  II. Scalar multiplication: $\alpha \vec{a}$ stretching of $\vec{a}$ by a factor of α (real number)

  III. Coordinates:

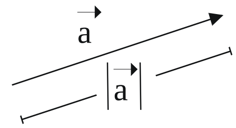
$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix},$$

$$\vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \\ a_3 \pm b_3 \end{pmatrix}, \alpha \vec{a} = \begin{pmatrix} \alpha a_1 \\ \alpha a_2 \\ \alpha a_3 \end{pmatrix}$$



IV. Modulus of vector $\vec{a}$

is its length $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$



- Dealing with ”Numbers” should be quite familiar, e.g. solving simple equations:
  $x^2 - 4x + 3 = 0$,
  $\sin(\pi x) = 0$
- Complex functions as vectors of a linear vector space
  - functions: $f(x)$ are vectors of linear vector space
  - scalar product $\langle f|g\rangle := \int_{-\infty}^{+\infty} f^*(x)g(x)dx$
  - only square integrable, but complex functions (x: real)
  - functions are parallel, orthonormal, angle between two functions can be calculated
  - sets of basic functions exist, projections can be calculated, ...
  - necessary for quantum mechanics
  - helpful for understanding of Fourier analysis, ...