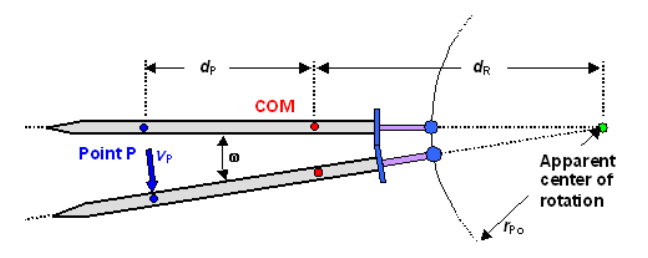
Angular velocity and Switching Systems

Illustration

This is an Illustration and not a science link because it just illustrates your thought processes. I'm sure that everything in here you could figure out yourself. Angela Merkel could (but possibly not D. Trump).

- We use the picture from the "Hitting Something" subchapter. A sword rotates around an apparent (or instantaneous) center of rotation. How fast does some point **P** somewhere on the blade move?



Movement described by a rotation around an apparent center of rotation

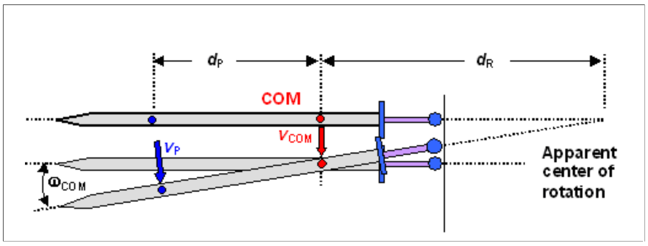
We assume of course a constant rotational speed or velocity. That means that the sword will finish a full circle in some time T_{cycle}. It thus makes $1/T_{cycle}$ = ν full rotations per time unit. As a unit for the rotational speed you thus could give the number of revolutions *per minute* and call that "***rpm***". You just as well could do the number of revolutions *per second* and then you would call that the **cycle frequency** ν with the unit "hertz" [Hz]. And yes, **1 Hz = 1/s**.

- So how fast does a point ***P*** travel that is found at some distance r_P from the apparent center of rotation? Since it travels on a perfect circle with radius r_P, it covers a distance equal to the circumference C_P of a circle with radius r_P for every revolution. C_P is equal to $2\pi r_P$ as even D.T. might know. Since we have ν rounds per second, the total distance covered in **1** second and thus the speed is $2\pi\nu \cdot r_P$.
  That gives us an idea: We can save a lot of writing if we don't use the cycle frequency $\nu_{cycle} = 1/T_{cycle}$ but something called the **circular frequency** (or circle frequency) $\omega = 2\pi\nu$.
  It is a simple as that. Using ω instead of ν or **1/T** or, God forbid, **rpm**, makes writing equations much more efficient. So everybody who is anybody uses the circular frequency and nothing else for rotational velocities.
- So how fast does a point ***P*** at some distance r_P from the apparent center of rotation travel? Here it is:

$$v_P = \omega \cdot r_P$$
$$= \omega \cdot (d_P + d_r)$$

for the point **P** shown

So far, so easy. But what happens if we do not describe the movement of the sword as a rotation around the apparent center of rotation (COM) but, in the way discussed at length in the backbone , as a combination of a pure translation for the center of mass plus a pure rotation around the center of mass. This is schematically shown below.



Movement described by a translation of, and a rotation around the COM

- The point *P* then moves with the speed of the COM which is $\mathbf{v_{COM}} = \omega \cdot \boldsymbol{d_r}$ plus the rotational speed for the rotation around the COM which is $\mathbf{v_{rot,\,COM}} = \omega_{COM} \cdot \boldsymbol{d_P}$. The circle frequency ω_{COM} for the rotation around the center of mass is identical to the one we had before since, as the picture shows, the same rotation angle applies, i.e. $\omega_{COM} = \omega$,
  This leaves us with

$$
\begin{aligned}
v_P &= \omega_{COM} \cdot d_P + \omega \cdot d_r \\
&= \omega \cdot (d_P + d_r)
\end{aligned}
$$

- Same as above. It also works for the kinetic energies $\boldsymbol{E_{kin}}$ They must the same in both cases. With **=** moment of inertia for rotation around the apparent center of rotation, and I_{COM} = moment of inertia for rotation around the COM, we get for the kinetic energy using the rotation around the apparent center of rotations:

$$
\begin{aligned}
E_{kin} &= \tfrac{1}{2}\ I_A \cdot \omega^2 \\
&= \tfrac{1}{2}\ (I_{COM} + md_R{}^2)\omega^2
\end{aligned}
$$

- We used the [parallal axis theorem]() for this; ***m*** is the mass of the sword.
  Calculating the kinetic energy for the COM systen we get

$$
\begin{aligned}
E_{kin} &= \tfrac{1}{2}\ (I_{COM}\, \omega^2 + mv^2_{COM}) \\
&= \tfrac{1}{2}\ (I_{COM} + md_R{}^2)\omega^2
\end{aligned}
$$

Just as it should be. I rest my case.