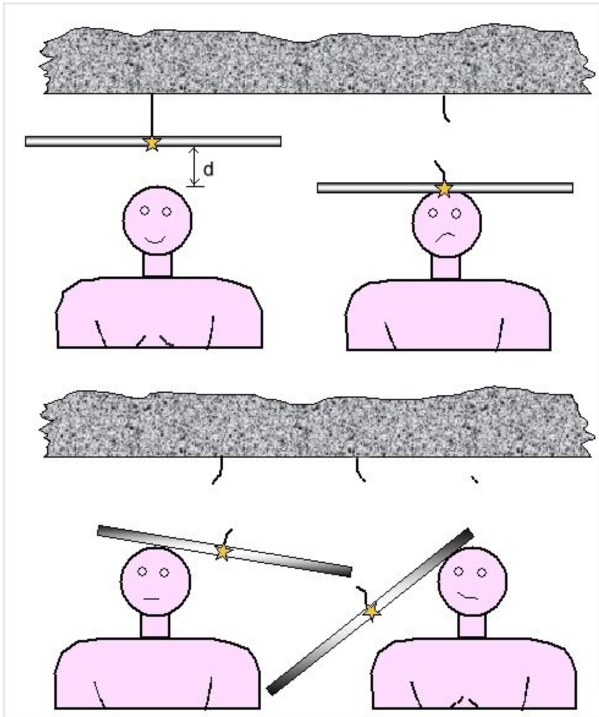
12.4 Wielding Your Sword

12.4.1 The Effective Mass or Apparent Inertia

What is the Effective Mass?

Finally! In this last subchapter we are going to wield a sword. First we will just swing it around to get a feeling of how it handles but then we will actually hit something or somebody with it. For example you. Right on the head. That will give us an idea of what I mean with the term "**effective mass**" or "**apparent inertia**" in the headline.
I'm going to use **Damocles sword** to hit you with. To make it easier for me (and you) I will use an idealized version of that, just a plain, uniform rod. Consider it dangling above your head in various positions relative to your head, and then the thin string rips. What will you experience?



Damocles sword hits you
Note that my drawings are gender mainstreamed

Let's start with the center of mass (COM) of the sword exactly above your head as in the upper pictures above. In the ideal physical world so beloved by basic physics teachers, the sword will hit you and come to rest in the dent on your head. That happens because your mass is much larger than that of the sword, and your head is rather soft. What you fell is painful since the total energy of the falling sword is transferred to your head. This energy is directly given by by the total mass of the sword. If you remember anything from your physics teaching, you know that it is simply mass ***m*** times the distance ***d*** between the hanging sword and your head (times the constant acceleration of gravity = 9.81 m / s^2). This potential energy of the sword relative to your head is converted to kinetic energy (one-half of mass times velocity squared), and that's what you feel.
Note that the sword is rather stupid, It doesn't know if its velocity just before impact is due to free fall in earth's gravity or because somebody is moving it with his hands.
Now let's consider what you experience if the swords COM is *not* lined up with your head as in the lower pictures above.

- The sword will hit you somewhere along its length and then tumble down rotating. What do you feel? An impact less painful then the first one. The pain decreases with the distance between the COM of the sword and the point where it hits you. The least pain is encountered if you are hit by its very end.

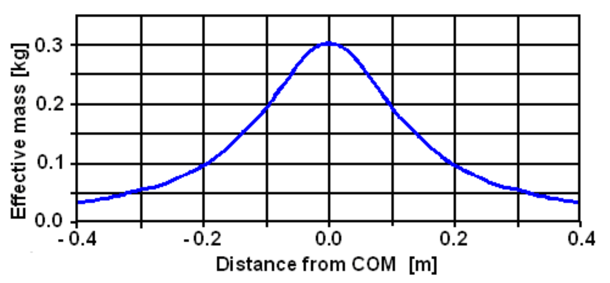
How can that be? After all, the energy of the sword at the moment of impact is the same in all scenarios? Yes, indeed - but not all of that energy is transferred to your head. Some kinetic energy remains in the sword because it keeps moving and some rotational energy is building up. It is not too difficult to calculate what exactly is happening for a simple uniform rod, a sword, an axe, or whatever. I'll do that in an science module but here I'll keep it simple

- And I have good news: There is an ingenious shortcut: Since any point of a falling object has the

And I have good news: There is an ingenious shortcut: Since any point of a falling object has the same velocity when it hits you, you can describe the different energies transferred to you by assigning an **effective mass** to the sword that depends on position along its length. Turner calls the effective mass "**apparent inertia**" or "**apparent mass**" of the sword, which is fine too. I'll use the term "effective mass" because that is something physicists are used to, it comes up, for example, in semiconductor physics all the time In the experiment with a uniform rod discussed above, we can deduce without any calculations that the effective mass must be equal to the real mass at the position of the center of mass, right in the center of the rod, and then it must decrease somehow towards both ends. That is exactly what it does if you run through the calculations: Here is what we get for a 0.3 kg sword:

Science Link

Effective Mass

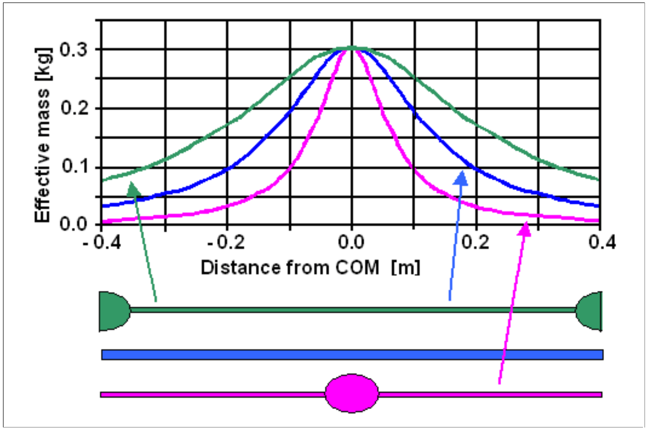


Effective mass distribution

What we also get is that the only necessary parameter of the object in question that we need to know is its *moment of inertia* relative to its center of mass (COM). That is rather amazing and allows many insights into the optimization of real swords, as we shall see.
I use the term "optimization" because we want to *optimize* as opposed to *maximize* properties. That involves going for compromises since measures that might be good for one property will turn out to be bad for another one. With the effective mass we have yet another key parameter - sort of a mixture of the mass and the moment of inertia - that will not only prove quite helpful but is essentialfor understanding what follows.

Applications of the Effective Mass Concept

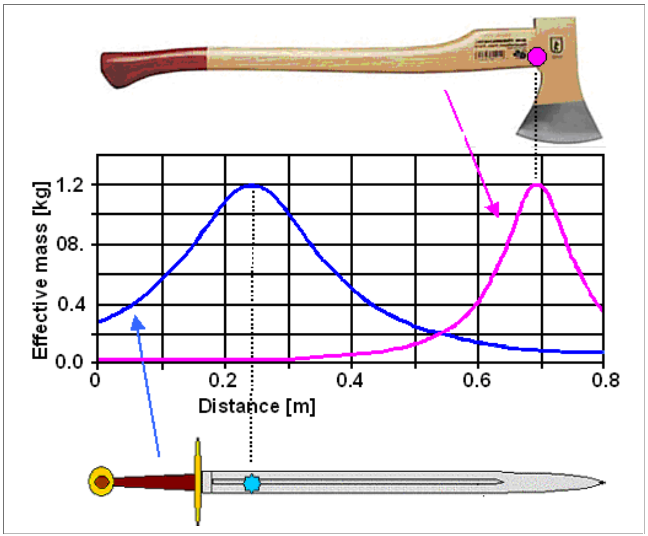
- First let's look at what happens of we change the moment of inertia of our rod from above without changing its weight. We might do that by putting a sphere containing most of the weight in the middle (while making the rod thinner to keep the mass constant) or two half-spheres at the ends as shown below. The first measure decrease the moment of inertia quite a bit, the second measure increases it.



Effective mass of three rods with the same mass (0.3 kg) but different mass distributions (as schematically indicated) and thus moments of inertia *I*

  - The effective mass curve responds simply by constricting or widening as shown for a rod weighing 0.3 kg. What you would feel in the Damocles sword experiment is also quite clear. The impact with the COM hitting your head is unchanged. It hurts the same for all three rods. Getting hit by the end of the rod is different, however. Far less pain with the low-***I*** rod. far more with the high-***I*** rod.

- Next we look at the effective mass curve of a real sword and an axe. We get something like this:



Effective mass of sword and axe

- The curves, by the way, are not actually calculated for the weapons shown but just give the idea qualitatively. What you perceive for sure is that you rather have the business end of a sword fall on you than the business end of axe. Its effective mass at this point is much higher than that of the sword and thus also the energy transmitted to your head.
  - Well - the sword is preferable to the axe only as long as both objects fall and thus hit you with the same velocity. If somebody swings these weapons, chances are that he can swing the sword much faster than the axe, so it hits you at higher velocity. And the energy, remember, goes with the mass and the *square* of the velocity! There is a reason why you might prefer your axe whenever you have all the time in the world to get it up to speed but go for your far more agile sword if there is high-speed fighting.

- What you want is an appreciable effective mass at the tip of your sword so its hits will be felt but without compromising on all the other points like manageable total mass, small moment of inertia for agility, and a percussion point (plus the important vibration nodes) at the right place.
  And now you start to see the magnitude of the problem we are facing when we want to optimize a sword.