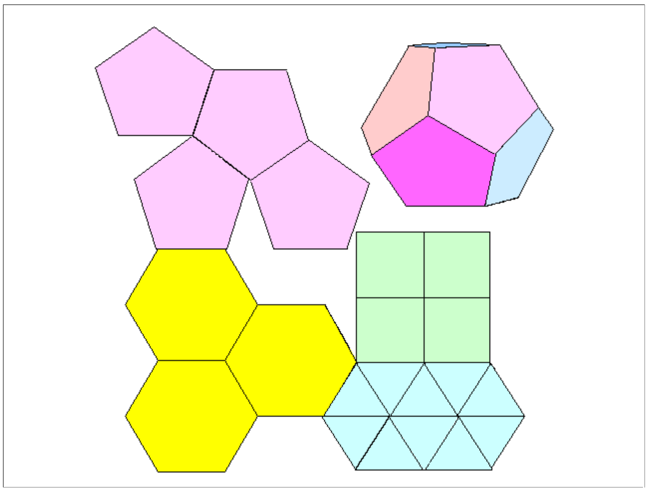
# Quasi Crystals

Advanced

## History and Basics

- You don't have to be a genius to realize that you can't completely tile your two-dimensional bathroom floor or walls with regular five-sided tiles or pentagons. Hexagons - yes. Equilateral triangles, squares, and rectangles - yes; see the figure below.
    - You don't have to be a genius either to realize that you cannot "tile" three-dimensional space (akin to filling out a volume) with regular five-sided bricks (called dodecahedrons) either. Rectangular bricks - yes. Tetrahedrons - yes. Others? Who knows. But dodecahedrons - definitely no.



**Tiling the plane. It is simply not possible with pentagons**

"Tiling" or bricking up space with dodecahedrons is simply not possible either. A dodecahedron is shown in the upper right corner.

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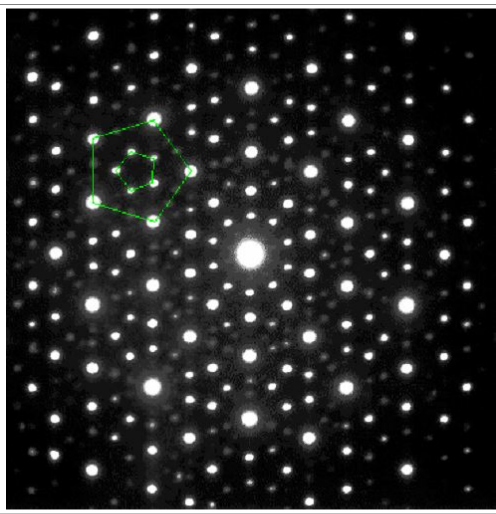
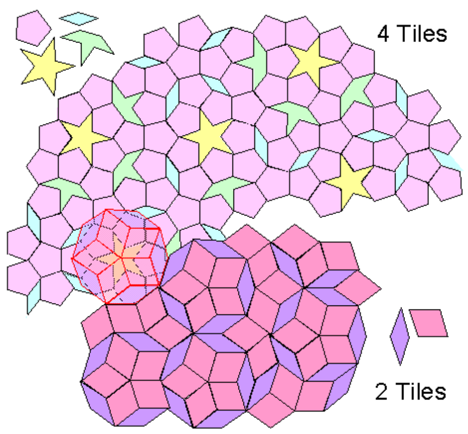
    - You don't have to be a genius either to know that the *diffraction pattern* of regular crystals therefore can *never* have a five-fold symmetry. But you need to know a bit about crystals and Fourier transformations though, or you must now look at this module.
- When in 1982 **Dan Shechtman** observed a diffraction pattern like the one below, he was flabbergasted, to put it mildly. His specimen was a rapidly cooled alloy of aluminum (Al) and Manganese (Mn), nothing particularly exciting. The presence of a diffraction pattern proved that the material was crystalline, i.e. with a regular arrangement of its atoms, and the 5 or 10 fold symmetry of the diffraction pattern proved that there must be an arrangement of building blocks that was simply impossible according to common and well justified believe.
    - Of course, nobody believed Shechtman at first. His boss suggested that he quit, extremely famous Nobel prize winner **Linus Pauling** pontificated that "there are no quasicrystals, only quasiscientists"; journals refused to print his papers. In 2011 Dan Shechtman was awarded the Nobel prize.



| **Diffraction pattern of a quasicrystal (Electron microscope)** |
|---|
| Note the "impossible" 10-fold rotational symmetry.<br>There are lots of pentagons, two are outlined in green. |

- The break-through occurred in 1984 after some open-minded scientists adopted Shechtman's results. It also became clear that the solution to the obvious problem was already at hand, since more mathematically inclined scientists had already dealt with the problem theoretically years before Shechtman's discovery.
  - In particular, famous **Roger Penrose** had shown in 1974 how to tile a plane in a way that provided regularity *and* a five-fold symmetry. What was lost was the usual translation symmetry of normal crystals, i.e. the property that nothing changes in the pattern if you just move it in the plane.
    The figure below illustrates this "Penrose tiling".

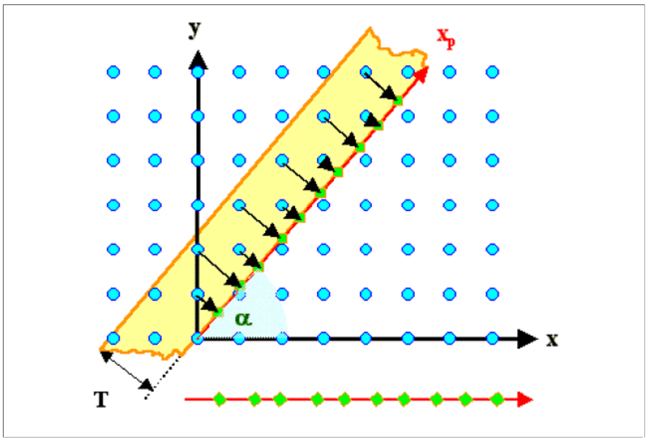



**Penrose tiling and quasicrystals.**

  - Penrose used 4 differently shaped tiles but you can get away with just two types as shown. Just connect the centres of the old tiles as illustrated in the overlapping part.
    There are a lot of five-fold structures in this tiling, *and* there is a lot of regularity.
    Now do the whole thing with properly chosen bricks in three dimensions, and you have the structure of the quasicrystals that Shechtman observed.
    Scientists have made a lot of quasiscrystals by now; even Mother Nature makes them. They have rather peculiar properties but we still have to find major applications for this new class of materials

## Boggling the Mind

- Now lets give the difficult part a quick look. First, we ask ourselves if, maybe, you could tile the plane in a quasicrystal pattern with just *one* tile? It is clear to you that this question has a defined answer: yes or no!
  - It is probably not so clear to you that this question belongs to the class of perfectly legitimate questions with a definite yes / no answer for which it is impossible to find the answer in a *systematic* way, i.e. by an *algorithm*.
    Mathematicians have proved that with mathematical rigor. You may happen to find the answer somehow, for example by lucky guessing, but you will never be able to program a computer to find the answer.
    So far, by the way, nobody has found the answer to the question above. It is rather unlikely that one tile will do, but who knows?
- Now to the really weird part. Quasicrystals actually result from perfectly legitimate *real* crystals with translation symmetry and everything - if you construct these crystals in *six* dimensions. Mathematicians have no problems to do that with equations, it just boggles the mind a bit to imagine it.
  - All you need to do is to construct a nice six-dimensional crystal. Then you project some of its lattice points onto our common three-dimensional space in some proper way. You can't imagine that? I can't either.
    But it's just like projecting three-dimensional objects onto two-dimensional space, just a bit more involved. Take a sphere, for example. Project it onto a plane and you have a circle for all projection geometries. Project a cube and you can get squares, rectangles and hexagons, depending on the projection geometry.
  - Let's do it in an even simpler way. We take a lattice in *two* dimensions and project it onto *one*-dimensional space, i.e. on a line. The figure below illustrates how it is done:




**Generating quasicrystals by projection from higher dimensions.**

- The general recipe is simple:
  - Produce a two-dimensional (six-dimensional) lattice / crystal as shown.
  - Draw a one-dimensional (three-dimensional) space into the two-dimensional (six-dimensional) one. Make sure that there is some irrational relation. A vector along the one-dimensional line might have the components [1, $(2)^{1/2}$], for example. The one-dimensional line then will never run across a lattice point because of this.
  - Project the lattice points found inside some one-dimensional (three-dimensional space) distance***T*** to the line onto the line (three-dimensional space).
  - You have produced a quasicrystal.
- Look at the sequence of projected points on the red line above. There is clearly some regularity but the pattern never repeats; it is a one-dimensional quasicrystal.
  If you have a lot of free time, you can now do the same thing for a three-dimensional crystal projected onto a two-dimensional space, i.e. a plane. Please send me the drawings after you are done.

Now you are thinking that this is pretty crazy but just what you would expect of the nerdiest of the nerds, the mathematicians. Surly, this stuff has no bearing on real quasicrystals that you can buy and touch?

- Not so. Lattice defects like dislocations, one should think, can only exist in real three-dimensional crystals. Amazingly enough, with an electron microscope, we see things that look like dislocations and behave like dislocations also in *quasi* crystals. These things are dislocations. But, as it turns out, only if we associate a *six* -dimensional vector with them, in contrast to dislocations in normal crystals, where a three-dimensional vector is sufficient.

If you now admit that Dan Shechtman got his Nobel prize for rather good reasons, I will rest my case.