4.5. Summary: Magnetic Materials

The **relative permeability** μ_r of a material "somehow" describes the interaction of magnetic (i.e. more or less all) materials and magnetic fields H, e.g. vial the equations ⇒

- **B** is the **magnetic flux density** or **magnetic induction**, sort of replacing H in the Maxwell equations whenever materials are encountered.

- **L** is the inductivity of a linear solenoid, or)coil or inductor) with length l, cross-sectional area A, and number of turns t, that is "filled" with a magnetic material with μ_r.

- **n** is *still* the index of refraction; a quantity that "somehow" describes how electromagnetic fields with extremely high frequency interact with matter.
  For all practical purposes, however, $\mu_r = 1$ for optical frequencies

$$B = \mu_o \cdot \mu_r \cdot H$$

$$L = \frac{\mu_0 \cdot \mu_r \cdot A \cdot w^2}{l}$$

$$n = (\epsilon_r \cdot \mu_r)^{\frac{1}{2}}$$

Magnetic fields inside magnetic materials polarize the material, meaning that the vector sum of magnetic dipoles inside the material is no longer zero.

- The decisive quantities are the *magnetic* dipole moment $\underline{m}$, a vector, and the *magnetic* Polarization J, a vector, too.

- Note: In contrast to dielectrics, we define an additional quantity, the **magnetization M** by simply including dividing J by μ_o.

- The magnetic dipoles to be polarized are either already present in the material (e.g. in **Fe, Ni or Co**, or more generally, in all **paramagnetic** materials, or are induced by the magnetic fields (e.g. in **diamagnetic** materials).

- The dimension of the magnetization $\underline{M}$ is **[A/m]**; i.e. the same as that of the magnetic field.

$$B = \mu_o \cdot H + J$$

$$\underline{J} = \mu_o \cdot \frac{\Sigma \underline{m}}{V}$$

$$M = \frac{J}{\mu_o}$$

The magnetic polarization J or the magnetization M are *not* given by some magnetic surface charge, because ⇒.

There is no such thing as a *magnetic monopole*, the (conceivable) counterpart of a negative or positive electric charge

The equivalent of "Ohm's law", linking current density to field strength in conductors is the *magnetic* Polarization law:

- The decisive material parameter is $\chi_{mag} = (\mu_r - 1)$ = **magnetic susceptibility**.

- The "classical" induction B and the magnetization are linked as shown. In essence, $\underline{M}$ only considers what happens in the material, while B looks at the total effect: material plus the field that induces the polarization.

$$M = (\mu_r - 1) \cdot H$$

$$M := \chi_{mag} \cdot H$$

$$B = \mu_o \cdot (H + M)$$

Magnetic polarization mechanisms are formally similar to dielectric polarization mechanisms, but the physics can be entirely different.

Atomic mechanisms of magnetization are not directly analogous to the dielectric case

Magnetic moments originate from:

- The intrinsic magnetic dipole moments **m** of elementary particles with spin is measured in units of the Bohr magneton m_{Bohr}.

- The magentic moment m^e of the electron is ⇒

- Electrons "orbiting" in an atom can be described as a current running in a circle thus causing a magnetic dipole moment; too

$$m_{Bohr} = \frac{h \cdot e}{4\pi \cdot m^*_e} = 9.27 \cdot 10^{-24} \text{ Am}^2$$

$$m^e = \frac{2 \cdot h \cdot e \cdot s}{4\pi \cdot m^*_e} = 2 \cdot s \cdot m_{Bohr} = \pm m_{Bohr}$$

The total magentic moment of an atom in a crystal (or just solid) is a (tricky to obtain) sum of all contributions from the electrons, and their orbits (including bonding orbitals etc.), it is either:

- **Zero** - we then have a **diamagmetic material**.

  Magnetic field induces dipoles, somewhat analogous to elctronic polarization in dielectrics. Always very weak effect (except for superconductors) Unimportant for technical purposes

- In the order of a few Bohr magnetons - we have a essentially a **paramagnetic material**.

  Magnetic field induces some order to dipoles; strictly analogous to "orientation polarization" of dielectrics. Always very weak effect Unimportant for technical purposes

In some **ferromagnetic** materials spontaneous ordering of magenetic moments occurs below the Curie (or Neél) temperature. The important familiess are

- • Ferromagnetic materials ⇑⇑⇑⇑⇑⇑⇑ large μ_r, **extremely important**.
  • Ferrimagnetic materials ⇑⇓⇑⇓⇑⇓⇑ still large μ_r, **very important**.
  • Antiferromagnetic materials ⇑⇓⇑⇓⇑⇓⇑ $\mu_r \approx 1$, unimportant

  Ferromagnetic materials: Fe, Ni, Co, their alloys "AlNiCo", Co_5Sm, $Co_{17}Sm_2$, "NdFeB"

There is characteristic temperatuer dependence of μ_r for all cases

Dia- and Paramagentic propertis of materials are of no consequence whatsoever for products of electrical engineering (or anything else!)

- Only their common denominator of being essentially "non-magnetic" is of interest (for a submarine, e.g., you want a non-magnetic steel)

- For research tools, however, these forms of magnitc behavious can be highly interesting ("paramagentic resonance")

Normal diamagnetic materials: $\chi_{dia} \approx -(10^{-5} - 10^{-7})$
Superconductors (= ideal diamagnets): $\chi_{SC} = -1$
Paramagnetic materials: $\chi_{para} \approx +10^{-3}$

Diamagnetism can be understood in a semiclassical (Bohr) model of the atoms as the response of the current ascribed to "circling" electrons to a changing magnetic field via classical induction ($\propto$ **dH/dt**).

- The net effect is a precession of the circling electron, i.e. the normal vector of its orbit plane circles around on the green cone. ⇒

- The "Lenz rule" ascertains that inductive effects oppose their source; diamagnetism thus weakens the magnetic field, $\chi_{\textbf{dia}} < \textbf{0}$ must apply.
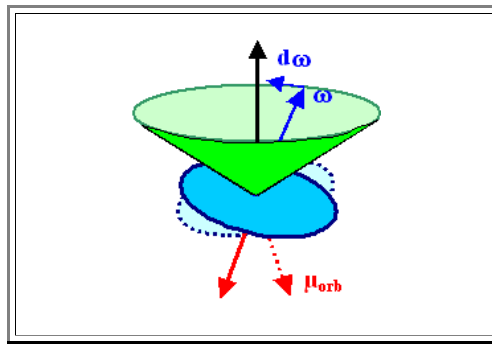
Running through the equations gives a result that predicts a very small effect. ⇒
A proper quantum mechanical treatment does not change this very much.

$$\chi_{\text{dia}} = - \frac{e^2 \cdot z \cdot \langle r \rangle^2}{6\, m^*_e} \cdot \rho_{\text{atom}} \approx -(10^{-5} - 10^{-7})$$

The formal treatment of paramagnetic materuials is mathematically completely identical to the case of orientation polarization

- The range of realistc β values (given by largest H technically possible) is even smaller than in the case of orientation polarization. This allows tp approximate **L(β)** by β**/3**; we obtain:

$$\chi_{\textbf{para}} = \frac{N \cdot m^2 \cdot \mu_0}{3kT}$$

- Insertig numbers we find that $\chi_{\textbf{para}}$ is indeed a number just slightly larger than **0**.

$$W(\varphi) = -\mu_0 \cdot \underline{m} \cdot \underline{H} = -\mu_0 \cdot m \cdot H \cdot \cos \varphi$$

Energy of magetic dipole in magnetic field

$$N[W(\varphi)] = c \cdot \exp -(W/kT) = c \cdot \exp \frac{m \cdot \mu_0 \cdot H \cdot \cos \varphi}{kT} = N(\varphi)$$

(Boltzmann) Distribution of dipoles on energy states

$$M = N \cdot m \cdot L(\beta)$$

$$\beta = \frac{\mu_0 \cdot m \cdot H}{kT}$$

Resulitn Magnetization with Langevin function **L(β)** and argument β

In ferromagnetic materials the magnetic moments of the atoms are "correlated" or lined-up, i.e. they are all pointing in the same direction

- The physical reason for this is a quantum-mechanical spin-spin interaction that has no simple classical analogue.

- However, exactly the same result - complete line-up - could be obtained, if the magnetic moments would feel a strong magnetic field.

- In the "mean field" approach or the "Weiss" approach to ferromagnetism, we simply assume such a magnetic field $H_{\textbf{Weiss}}$ to be the cause for the line-up of the magnetic moments. This allows to treat ferromagnetism as a "special" case of paramagnetism, or more generally, "orientation polarization".
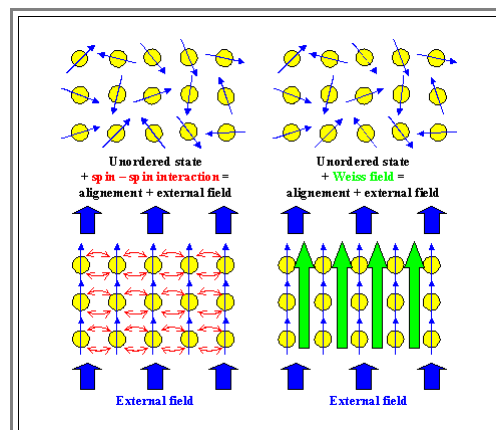
For the magnetization we obtain ⇒

- The term $w \cdot J$ describes the Weiss field via $H_{\textbf{loc}} = H_{\textbf{ext}} + w \cdot J$; the Weiss factor w is the decisive (and unknown) parameter of this approach.

$$J = N \cdot m \cdot \mu_0 \cdot L(\beta) = N \cdot m \cdot \mu_0 \cdot L \left(\frac{m \cdot \mu_0 \cdot (H + w \cdot J)}{kT} \right)$$

- Unfortunately the resulting equation for **J**, the quantity we are after, cannot be analytically solved, i.e. written down in a closed way.

- Graphical solutions are easy, however ⇒

  - From this, and with the usual approximation for the Langevin function for small arguments, we get all the major ferromagnetic properties, e.g.

- Saturation field strength.
- Curie temperature T_C.

$$T_C = \frac{N \cdot m^2 \cdot \mu_0^2 \cdot w}{3k}$$

- Paramagnetic behavior above the Curie temperature.
- Strength of spin-spin interaction via determining w from T_C.

  - As it turns out, the Weiss field would have to be far stronger than what is technically achievable - in other words, the spin-spin interaction can be exceedingly strong!
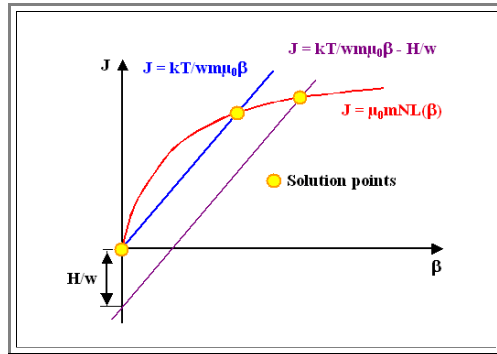
- In single crystals it must be expected that the alignments of the magnetic moments of the atom has some preferred crystallographic direction, the "easy" direction.

Easy directions:
Fe (bcc) <100>
Ni (fcc) <111>
Co (hcp) <001> (c-direction)

- A single crystal of a ferromagnetic material with all magnetic moments aligned in its easy direction would carry a high energy because:

 - It would have a large external magnetic field, carrying field energy.

- In order to reduce this field energy (and other energy terms not important here), magnetic domains are formed ⇒. But the energy gained has to be "payed" for by:
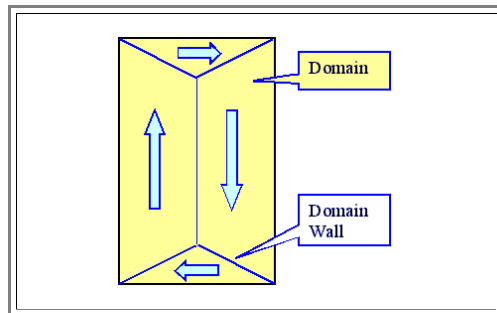
  - Energy of the domain walls = planar "defects" in the magnetization structure. It follows: Many small domains —> optimal field reduction —> large domain wall energy "price".

  - In polycrystals the easy direction changes from grain to grain, the domain structure has to account for this.

  - In all ferromagnetic materials the effect of magnetostriction (elastic deformation tied to direction of magnetization) induces elastic energy, which has to be minimized by producing a optimal domain structure.
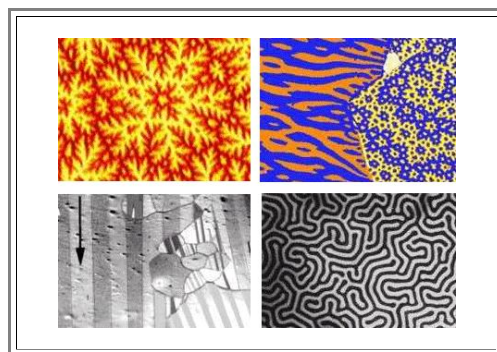

- The domain structures observed thus follows simple principles but can be fantastically complicated in reality ⇒.

For ferromagnetic materials in an external magnetic field, energy can be gained by increasing the total volume of domains with magnetization as parallel as possible to the external field - at the expense of unfavorably oriented domains.

- Domain walls must move for this, but domain wall movement is hindered by defects because of the elastic interaction of magnetostriction with the strain field of defects.
- Magnetization curves and hystereses curves result ⇒, the shape of which can be tailored by "defect engineering".

Domain walls (mostly) come in two varieties:

- Bloch walls, usually found in bulk materials.
- Neél walls, usually found in thin films.
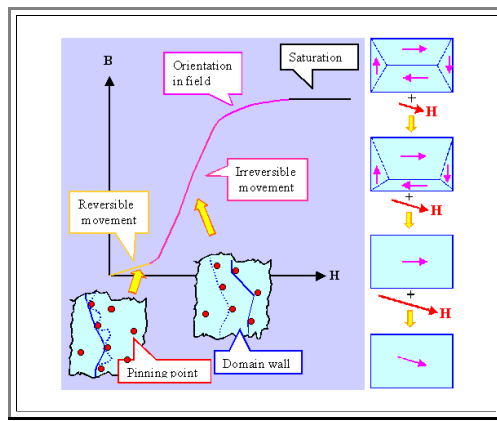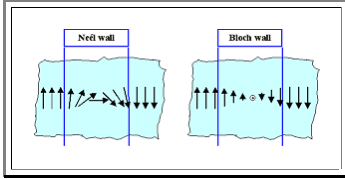
Depending on the shape of the hystereses curve (and described by the values of the remanence M_R and the coercivity H_C, we distinguish hard and soft magnets ⇒.
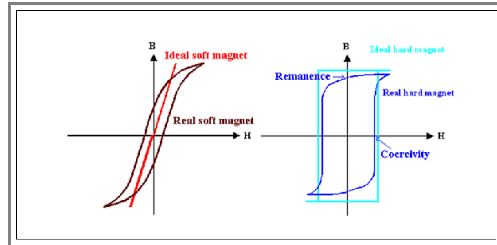
Tailoring the properties of the hystereses curve is important because magnetic losses and the frequency behavior is also tied to the hystereses and the mechanisms behind it.

- Magnetic losses contain the (trivial) eddy current losses (proportional to the conductivity and the square of the frequency) and the (not-so-trivial) losses proportional to the area contained in the hystereses loop times the frequency.
- The latter loss mechanism simply occurs because it needs work to move domain walls.

It also needs time to move domain walls, the frequency response of ferromagnetic materials is therefore always rather bad - most materials will not respond anymore at frequencies far below **GHz**.

Uses of ferromagnetic materials may be sorted according to:

- Soft magnets; e.g. **Fe** - alloys

    - **Everything profiting from an "iron core": Transformers, Motors, Inductances, ...**
    - **Shielding magnetic fields.**

- Hard magnets; e.g. metal oxides or "strange" compounds.

    - **Permanent magnets for loudspeakers, sensors, ...**
    - **Data storage (Magnetic tape, Magnetic disc drives, ...**

Even so we have essentially only **Fe**, **Ni** and **Co** (+ **Cr**, **O** and **Mn** in compounds) to work with, innumerable magnetic materials with optimized properties have been developed.

- New complex materials (including "nano"materials) are needed and developed all the time.

Data storage provides a large *impetus* to magnetic material development and to employing new effects like "**GMR**"; giant magneto resistance; a purely quantum mechanical effect.

Questionaire

Multiple Choice questions to all of 4