4.2.2 Paramagnetism

The treatment of paramagnetism in the most simple way is exactly identical to the treatment of orientation polarization. All you have to do is to replace the *electric dipoles* by magnetic dipoles, which we call *magnetic moments*.

- We have permanent dipole moments in the material, they have no or negligible interaction between them, and they are free to point in any direction *even in solids*.
- This is a major difference to electrical dipole moments which can *only* rotate if the whole atom or molecule rotates; i.e. only in liquids. This is why the treatment of magnetic materials focusses on ferromagnetic materials and why the underlying symmetry of the math is not so obvious in real materials.
- In an external magnetic field the magnetic dipole moments have a tendency to orient themselves into the field direction, but this tendency is opposed by the thermal energy, or better entropy of the system.

Using exactly the same line of argument as in the case of orientation polarization, we have for the potential energy W of a magnetic moment (or dipole) m in a magnetic field H

$$W(\varphi) = -\mu_0 \cdot \underline{m} \cdot \underline{H} = -\mu_0 \cdot m \cdot H \cdot \cos\varphi$$

- With φ = angle between $\underline{H}$ and $\underline{m}$.

In *thermal equilibrium*, the number of magnetic moments with the energy W will be $N[W(\varphi)]$, and that number is once more given by the Boltzmann factor:

$$N[W(\varphi)] = c \cdot \exp -(W/kT) = c \cdot \exp \frac{m \cdot \mu_0 \cdot H \cdot \cos\varphi}{kT} = N(\varphi)$$

- As befoere, c is some as yet undetermined constant.

As before, we have to take the component in field direction of all the moments having the same angle with H and integrate that over the unit sphere. The result for the induced magnetization m_{ind} and the total magnetization M is the same as before for the induced dielectric dipole moment:

$$m_{ind} = m \cdot \left(\coth\beta - \frac{1}{\beta}\right)$$

$$M = N \cdot m \cdot L(\beta)$$

$$\beta = \frac{\mu_0 \cdot m \cdot H}{kT}$$

- With **L(β) = Langevin function = cothβ – 1/β**

The only interesting point is the *magnitude* of β. In the case of the orientation polarization it was $\leq$ **1** and we could use a simple approximation for the Langevin function.

- We know that m will be of the order of magnitude of **1 Bohr magneton**. For a rather large magnetic field strength of $5 \cdot 10^6$ **A/m**, we obtain as an estimate for an upper limit $\beta = 1.4 \cdot 10^{-2}$, meaning that the range of β is even smaller as in the case of the electrical dipoles.
- We are thus justified to use the simple approximation **L(β) = β/3** and obtain

$$M = N \cdot m \cdot (\beta/3) = \frac{N \cdot m^2 \cdot \mu_0 \cdot H}{3kT}$$

- The paramagnetic susceptibility $\chi = M/H$, finally, is

$$\chi_{para} = \frac{N \cdot m^2 \cdot \mu_0}{3kT}$$

- Plugging in some typical numbers (A Bohr magneton for ***m*** and typical densities), we obtain $\chi_{para} \approx +10^{-3}$; i.e. an *exceedingly small* effect, but with certain characteristics that will carry over to ferromagnetic materials:
  - There is a strong temperature dependence and it follows the "**Curie law**":

$$\chi_{para} = \frac{\text{const}}{T}$$

  - Since ferromagnets of all types turn into paramagnets above the Curie temperature T_C, we may simply expand Curies law for this case to

$$\chi_{ferro}(T > T_C) = \frac{\text{const*}}{T - T_C}$$

- In summary, paramagnetism, stemming from some (small) average alignement up of permanent magnetic dipoles associated with the atoms of the material, *is of no (electro)technical consequence*. It is, however, important for analytical purposes called "**Electron spin resonance**" (***ESR***) techniques.
- There are other types of paramagnetism, too. Most important is, e.g., the **paramagnetism of the free electron gas**. Here we have magnetic moments associated with spins of electrons, but in a *mobile* way **-** they are not fixed at the location of the atoms
  - But as it turns out, other kinds of paramagnetism (or more precisely: calculations taking into account that magnetic moments of atoms can not assume any orientation but only sone quantized ones) do not change the general picture: *Paramagnetism is a weak effect*.