4.2 Dia- and Paramagnetism

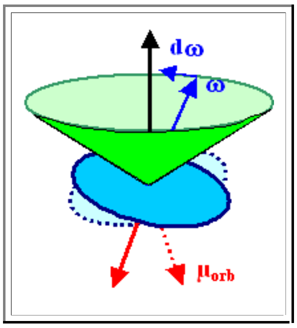
4.2.1 Diamagnetism

What is it Used for?

- It is customary in textbooks of electronic materials to treat dia- and paramagnetism in considerable detail. Considering that there is not a *single* practical case in electrical engineering where it is of any interest if a material is dia- or paramagnetic, there are only two justifications for doing this:
  - Dia- and paramagnetism lend themselves to *calculations* (and engineers like to calculate things).
  - It helps to *understand* the phenomena of magnetism in general, especially the quantum mechanical aspects of it.
- In this script we are going to keep the treatment of dia- and paramagnetism at a minimal level.

Diamagnetism - the Essentials

- The first thing to note about diamagnetism is that *all* atoms and therefore *all* materials show diamagnetic behavior.
  - Diamagnetism thus is always superimposed on all other forms of magnetism. Since it is a small effect, it is hardly noticed, however.
  - Diamagnetism results because all matter contains electrons - either "orbiting" the nuclei as in insulators or in the valence band (and lower bands) of semiconductors, or being "free", e.g. in metals or in the conduction band of semiconductors. All these electrons can respond to a (changing) magnetic field. Here we will only look at the (much simplified) case of a *bound electron orbiting a nucleus in a circular orbit*.
- The basic response of an *orbiting* electron to a changing magnetic field is a **precession** of the orbit, i.e. the polar vector describing the orbit now moves in a circle around the magnetic field vector ***H***:



  - The angular vector ω characterizing the blue orbit of the electron will experience a force from the (changing) magnetic field that forces it into a circular movement on the green cone.
  - Why do we emphasize "changing" magnetic fields? Because there is no way to bring matter into a magnetic field without changing it - either be switching it on or by moving the material into the field.
- What exactly happens to the *orbiting* electron? The reasoning given below follows the semi-classical approach contained within Bohr's atomic model. It gives essentially the right results *(in cgs units!)*.
  - The changing magnetic field, **d*H*/d*t***, generates a force ***F*** on the orbiting electron via inducing a voltage and thus an electrical field ***E***. We can always express this as

$$\boldsymbol{F} = m^*_e \cdot \boldsymbol{a} = m^*_e \cdot \frac{dv}{dt} := e \cdot \boldsymbol{E}$$

  - With ***a*** = acceleration = $dv/dt = e \cdot E/m^*_e$.
- Since **d*H*/d*t*** primarily induces a voltage ***V***, we have to express the field strength ***E*** in terms of the induced voltage ***V***. Since the electron is orbiting and experiences the voltage during *one* orbit, we can write:

$$E = \frac{V}{L}$$

  - With ***L*** = length of orbit = $2\pi \cdot r$, and ***r*** = radius of orbit.
  - ***V*** is given by the basic equations of induction, it is

$$V = -\frac{d\Phi}{dt}$$

- With Φ = magnetic flux **=** $\boldsymbol{H \cdot A}$; and $\boldsymbol{A}$ **=** area of orbit **=** $\pi \cdot r^2$. The *minus sign* is important, it says that the *effect* of a changing magnetic fields will be opposing the *cause* in accordance with **Lenz's law**.

Putting everything together we obtain

$$\frac{dv}{dt} = \frac{e \cdot E}{m^*_e} = \frac{V \cdot e}{L \cdot m^*_e} = -\frac{e \cdot r}{2\, m^*_e} \cdot \frac{dH}{dt}$$

The total change in **v** will be given by integrating:

$$\Delta v = \int_{v_1}^{v_2} dv = -\frac{e \cdot r}{2m^*_e} \cdot \int_0^H dH = -\frac{e \cdot r \cdot H}{2\, m^*_e}$$

The magnetic moment $\boldsymbol{m_{orb}}$ of the undisturbed electron was $\boldsymbol{m_{orb}} = ½ \cdot e \cdot v \cdot r$

- By changing **v** by Δ**v**, we change $\boldsymbol{m_{orb}}$ by $\Delta \boldsymbol{m_{orb}}$, and obtain

$$\Delta m_{orb} = \frac{e \cdot r \cdot \Delta v}{2} = -\frac{e^2 \cdot r^2 \cdot H}{4m^*_e}$$

That is more or less the *equation for diamagnetism* in the primitive electron orbit model.

- What comes next is to take into account that the magnetic field does not have to be perpendicular to the orbit plane and that there are many electrons. We have to add up the single electrons and average the various effects.
- Averaging over all possible directions of $\boldsymbol{H}$ (taking into account that a field in the plane of the orbit produces zero effect) yields for the *average* induced magnetic moment almost the same formula:

$$\Delta m_{orb} = \langle \Delta m_{orb} \rangle = -\frac{e^2 \cdot \langle r \rangle^2 \cdot H}{6m^*_e}$$

- $\langle r \rangle$ denotes that we average over the orbit radii at the same time

Considering that not just *one*, but all $\boldsymbol{z}$ electrons of an atom participate, we get the final formula:

$$\Delta m = \langle \Delta m_{orb} \rangle = -\frac{e^2 \cdot z \cdot r^2 \cdot H}{6\, m^*_e}$$

The additional magnetization $\boldsymbol{M}$ caused by $\Delta \boldsymbol{m}$ is *all the magnetization there is for diamagnets*; we thus we can drop the Δ and get

$$M_{Dia} = \frac{\langle \Delta m \rangle}{V}$$

With the definition for the magnetic susceptibility $\chi = M/H$ we finally obtain for the relevant material parameter for diamagnetism

$$\chi_{dia} = -\frac{e^2 \cdot z \cdot \langle r \rangle^2}{6\, m^*_e \cdot V} = -\frac{e^2 \cdot z \cdot \langle r \rangle^2}{6\, m^*_e} \cdot \rho_{atom}$$

- With ρ_{atom} = number of atoms per unit volume

Plugging in numbers will yield χ values around **– (10^{-5} - 10^{-7})** in good agreement with experimental values.