3.7.3 Using the Complex Index of Refraction

- Lets look at the physical meaning of **n** and κ, i.e. the real and complex part of the complex index of refraction, by looking at an electromagnetic wave traveling through a medium with such an index.
  - For that we simply use the general formula for the electrical field strength ***E*** of an electromagnetic wave traveling in a medium with refractive index **n***. For simplicities sake, we do it one-dimensional in the ***x***-direction (and use the index "**x**" only in the first equation). In the most general terms we have

$$E_x = E_{0,x} \cdot \exp i \cdot (k_x \cdot x - \omega \cdot t)$$

  - With k_x = component of the wave vector in **x**-direction = $k = 2\pi/\lambda$, ω = circular frequency = $2\pi\nu$.
- No index of refraction in the formulas; but we know (it is hoped), what to do. We must introduce the velocity **v** of the elecromagnetic wave in the material and use the relation between frequency, wavelength, and velocity to get rid of ***k*** or λ, respectively.
  - In other words, we use

$$v = \frac{c}{n^*} \qquad v = \nu \cdot \lambda$$

$$k = \frac{2\pi}{\lambda} = \frac{\omega \cdot n^*}{c}$$

  - Of course, **c** is the speed of light in vacuum. Insertion yields

$$E_x = E_{0,x} \cdot \exp i \cdot \left(\frac{\omega \cdot n^*}{c} \cdot x - \omega \cdot t\right) = E_{0,x} \cdot \exp i \cdot \left(\frac{\omega \cdot (n + i \cdot \kappa)}{c} \cdot x - \omega \cdot t\right)$$

$$E_x = E_{0,x} \cdot \exp \cdot \left(\frac{i \cdot \omega \cdot n \cdot x}{c} - \frac{\omega \cdot \kappa \cdot x}{c} - i \cdot \omega \cdot t\right)$$

- The red expression is nothing but the wavevector, so we get a rather simple result:

$$E_x = \exp - \frac{\omega \cdot \kappa \cdot x}{c} \cdot \exp[i \cdot (k_x \cdot x - \omega \cdot t)]$$

- In words that means: if we use a complex index of refraction, the propagation of electromagnetic waves in a material is whatever it would be for a simple *real* index of refractions times a *damping factor* that decreases the amplitude exponentially as a function of ***x***.
  - Obviously, at a depth often called absorption length or penetration depth $W = c/\omega \cdot \kappa$, the intensity decreased by a factor **1/e**.
  - The imaginary part κ of the complex index of refraction thus describes rather directly the attenuation of electromagnetic waves in the material considered. It is known as **damping constant**, **attenuation index**, **extinction coefficient**, or (rather misleading) *absorption constant*. Misleading, because an absorption constant is usually the α in some exponential decay law of the form $I = I_0 \cdot \exp - \alpha \cdot x$.
  - Note: Words like "constant", "index", or "coefficient" are also misleading - because κ is not constant, but depends on the frequency just as much as the real and imaginary part of the dielectric function.

(Should be continued but won't)