## 3.7.3 Using the Complex Index of Refraction

Lets look at the physical meaning of **n** and  $\kappa$ , i.e. the real and complex part of the complex index of refraction, by looking at an electromagnetic wave traveling through a medium with such an index.

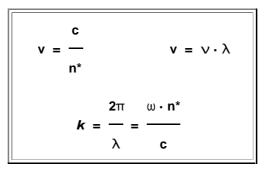
For that we simply use the general formula for the electrical field strength *E* of an electromagnetic wave traveling in a medium with refractive index n\*. For simplicities sake, we do it one-dimensional in the *x*-direction (and use the index "*x*" only in the first equation). In the most general terms we have

$$E_{\mathbf{X}} = E_{\mathbf{0}, \mathbf{X}} \cdot \exp \mathbf{i} \cdot (\mathbf{k}_{\mathbf{X}} \cdot \mathbf{x} - \mathbf{\omega} \cdot \mathbf{t})$$

With  $k_x$  = component of the wave vector in x-direction = k = 2π/λ, ω = circular frequency = 2πν.

No index of refraction in the formulas; but we know (it is hoped), what to do. We must introduce the velocity  $\mathbf{v}$  of the electromagnetic wave in the material and use the relation between frequency, wavelength, and velocity to get rid of  $\mathbf{k}$  or  $\lambda$ , respectively.

In other words, we use



Of course, **c** is the speed of light in vacuum. Insertion yields

$$E_{\mathbf{x}} = E_{0, \mathbf{x}} \cdot \exp \mathbf{i} \cdot \left(\frac{\omega \cdot \mathbf{n}^{*}}{c} \cdot \mathbf{x} - \omega \cdot \mathbf{t}\right) = E_{0, \mathbf{x}} \cdot \exp \mathbf{i} \cdot \left(\frac{\omega \cdot (\mathbf{n} + \mathbf{i} \cdot \mathbf{\kappa})}{c} \cdot \mathbf{x} - \omega \cdot \mathbf{t}\right)$$
$$E_{\mathbf{x}} = E_{0, \mathbf{x}} \cdot \exp \cdot \left(\frac{\mathbf{i} \cdot \omega \cdot \mathbf{n} \cdot \mathbf{x}}{c} - \frac{\omega \cdot \mathbf{\kappa} \cdot \mathbf{x}}{c} - \mathbf{i} \cdot \omega \cdot \mathbf{t}\right)$$

The red expression is nothing but the wavevector, so we get a rather simple result:

$$E_{\mathbf{x}} = \exp - \frac{\boldsymbol{\omega} \cdot \boldsymbol{\kappa} \cdot \boldsymbol{x}}{\mathbf{c}} \cdot \exp[\mathbf{i} \cdot (\boldsymbol{k}_{\mathbf{x}} \cdot \boldsymbol{x} - \boldsymbol{\omega} \cdot \mathbf{t})]$$

In words that means: if we use a complex index of refraction, the propagation of electromagnetic waves in a material is whatever it would be for a simple *real* index of refractions times a *damping factor* that decreases the amplitude exponentially as a function of *x*.

Obviously, at a depth often called absorption length or penetration depth W = c/ω · κ, the intensity decreased by a factor 1/e.

The imaginary part κ of the complex index of refraction thus describes rather directly the attenuation of electromagnetic waves in the material considered. It is known as **damping constant**, **attenuation index**, **extinction coefficient**, or (rather misleading) *absorption constant*. Misleading, because an absorption constant is usually the α in some exponential decay law of the form *I* = *I*<sub>0</sub> • exp – α • x.

Note: Words like "constant", "index", or "coefficient" are also misleading - because κ is not constant, but depends on the frequency just as much as the real and imaginary part of the dielectric function.

(Should be continued but won't)