3.7.2 The Complex Index of Refraction

In looking in detail at the polarization of dielectrics, we switched from a simple dielectric contant ϵ_r to a dielectric function $\epsilon_r(\omega) = \epsilon' + i\epsilon''$. This, after some getting used to, makes life much easier and provides for new insights not easily obtainable otherwise.

- We now do exactly the same thing for the index of refraction, i.e. we replace n by a **complex index of refraction** n^*.

$$n^* = n + i \cdot \kappa$$

- We use the old symbol n for the real part instead of n' and κ instead of n'', but that is simply to keep with tradition.
- With the dielectric *constant* and a *constant* index of refraction we had the basic relation ,

$$n^2 = \epsilon_r$$

- We simply use this relation now for defining the *complex* index of refraction. This gives us

$$(n + i\kappa)^2 = \epsilon' + i \cdot \epsilon''$$

- With $n = n(\omega)$; $\kappa = \kappa(\omega)$, since ϵ' and ϵ'' are frequency dependent as discussed before.

Re-arranging for n and κ yields somewhat unwieldy equations:

$$n^2 = \frac{1}{2}\left(\left(\epsilon'^2 + \epsilon''^2\right)^{1/2} + \epsilon'\right)$$

$$\kappa^2 = \frac{1}{2}\left(\left(\epsilon'^2 + \epsilon''^2\right)^{1/2} - \epsilon'\right)$$

Anyway - That is all. *We now have optics covered*. An example of an real complex index of refraction is shown in the link.

- So lets see how it works and what κ, the so far unspecified imaginary part of n_{com}, will give us.

First, lets get some easier formula. In order to do this, we remember that ϵ'' was connected to the conductivity of the material and express ϵ'' in terms of the (total) conductivity as

$$\epsilon'' = \frac{\sigma_{DK}}{\epsilon_0 \cdot \omega}$$

- Note that in contrast to the definition of ϵ'' given before in the context of the dielectric function, we have an ϵ_0 in the ϵ'' part. We had, for the sake of simplicity, made a convention that the ϵ in the dielectric function contain the ϵ_0, but here it more convenient to write it out, because then $\epsilon' = \epsilon_0 \cdot \epsilon_r$ is reduced to ϵ_r and directly related to the "simple" index of refraction n
- Using that in the expression $(n + i\kappa)^2$ gives

$$(n + i\kappa)^2 = n^2 - \kappa^2 + i \cdot 2n\kappa = \epsilon' + i \cdot \frac{\sigma_{DK}}{\epsilon_0 \cdot \omega}$$

- We have a complex number on both sides of the equality sign, and this demands that the real and imaginary parts must be the same on both sides, i.e.

$$n^2 - \kappa^2 = \epsilon'$$

$$n\kappa = \frac{\sigma_{DK}}{2\epsilon_0\omega}$$

- Separating ***n*** and **κ** finally gives

$$n^2 = \frac{1}{2}\left(\epsilon' + \left(\epsilon'^{\,2} + \frac{\sigma_{DK}{}^2}{4\epsilon_0{}^2\omega^{\,2}}\right)^{1/2}\right)$$

$$\kappa^2 = \frac{1}{2}\left(-\epsilon' + \left(\epsilon'^{\,2} + \frac{\sigma_{DK}{}^2}{4\epsilon_0{}^2\,\omega^2}\right)^{1/2}\right)$$

- Similar to what we had above, but now with basic quantities like the "dielectric constant" $\epsilon' = \epsilon_r$ and the conductivity $\sigma_{DK}{}^2$.

The equations above go beyond just describing the optical properties of (perfect) dielectrics because we can include all kinds of conduction mechanisms into σ, and all kinds of polarization mechanisms into **ϵ'**.

- We can even use these equations for things like the reflectivity of metals, as we shall see.

Keeping in mind that typical ***n***'s in the visible region are somewhere between **1.5 - 2.5** (***n*** ≈ **2.5** for diamond is one of the higher values as your girl friend knows), we can draw a few quick conclusions: From the simple but coupled equations for ***n*** and **κ** follows:

- **κ** should be rather small for "common" optical materials, otherwise our old relation of $n = (\epsilon_r)^{1/2}$ would be not good.
- **κ** should be rather small for "common" optical materials, because optical materials are commonly insulators, i.e. $\sigma_{DK} \approx 0$ applies.
- For $\sigma_{DK} = 0$ (and, as we would assume as a matter of course, $\epsilon_r > 0$) we obtain immediately $n = (\epsilon_r)^{1/2}$ and $\kappa = 0$ - the old-fashioned simple relation between just ϵ_r and ***n***.
- For large σ_{DK} values, both ***n*** and **κ** will become large. We don't know yet what **κ** means in physical terms, but very large ***n*** simply mean that the intensity of the reflected beam approaches **100 %**. Light that hits a good conductor thus will get reflected - well, that is exactly what happens between light and (polished) metals, as we know from everyday experience.

But now we must look at some problems that can be solved with the complex index of refraction in order to understand what it encodes.