

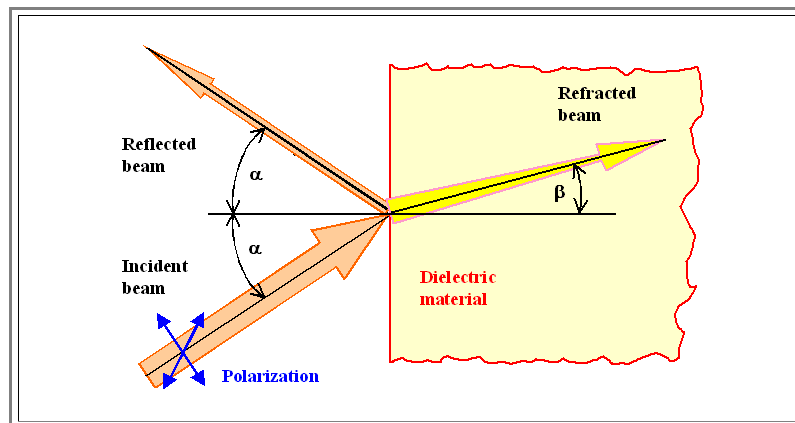
## 3.7 Dielectrics and Optics

### 3.7.1 Basics

- ▶ This subchapter can easily be turned into a whole lecture course, so it is impossible to derive all the interesting relations *and* to cover anything in depth. This subchapter therefore just tries to give a strong flavor of the topic.
- ▶ We know, *of course*, that the **index of refraction**  $n$  of a non-magnetic material is linked to the dielectric constant  $\epsilon_r$  via a simple relation, which is a rather direct result of the [Maxwell equations](#).

$$n = (\epsilon_r)^{1/2}$$

- ▶ But in learning about the origin of the dielectric constant, we have progressed from a simple constant  $\epsilon_r$  to a complex **dielectric function** with frequency dependent real and imaginary parts.
  - What happens to  $n$  then? How do we transfer the wealth of additional information contained in the *dielectric function* to optical properties, which are to a large degree encoded in the *index of refraction*?
- ▶ Well, you probably guessed it: We switch to a **complex index of refraction**!
  - But before we do this, let's ask ourselves what we actually want to find out. What are the optical properties that we like to know and that are *not* contained in a simple index of refraction?
  - Lets look at the *paradigmatic* experiment in optics and see what we *should* know, what we *already* know, and what we *do not* yet know.



- ▶ What we have is an electromagnetic wave, an *incident beam* (traveling in vacuum to keep things easy), which impinges on our dielectric material. As a result we obtain a *reflected beam* traveling in vacuum and a *refracted beam* which travels through the material. What do we know about the three beams?
  - The *incident beam* is characterized by its wavelength  $\lambda_i$ , its frequency  $\nu_i$  and its velocity  $c_0$ , the direction of its **polarization** in some coordinate system of our choice, and the arbitrary angle of incidence  $\alpha$ . We know, it is hoped, the simple **dispersion relation** for vacuum.

$$c_0 = \nu_i \cdot \lambda_i$$

- $c_0$  is, of course, the velocity of light in vacuum, an absolute constant of nature.
- ▶ The *incident beam* also has a certain *amplitude* of the electric field (and of the magnetic field, of course) which we call  $E_0$ . The *intensity*  $I_i$  of the light that the incident beams embodies, i.e. the energy flow, is proportional to  $E_0^2$  - never mix up the two!
- ▶ The *reflected beam* follows one of the basic laws of optics, i.e. **angle of incidence = angle of emergence**, and its wavelength, frequency and magnitude of velocity are identical to that of the incident beam.
  - What we do *not know* is its *amplitude* and its *polarization*, and these two quantities must somehow depend on the properties of the incident beam *and* the properties of the dielectric.
- ▶ If we now consider the *refracted beam*, we know that it travels under an angle  $\beta$ , has the same frequency as the incident beam, but a wavelength  $\lambda_d$  and a velocity  $c$  that is different from  $\lambda_i$  and  $c_0$ .
  - Moreover, we must expect that it is *damped* or *attenuated*, i.e. that its amplitude decreases as a function of penetration depth (this is indicated by decreasing thickness of the arrow above). All parameters of the refracted beam may depend on the polarization of the incident beam.
  - Again, *basic* optics teaches that there are some simple relations. We have

|   |                                 |
|---|---------------------------------|
| $\frac{\sin \alpha}{\sin \beta} = n$      | <b>Snellius law</b>             |
| $n = \frac{c_0}{c}$                       | <b>From Maxwell equations</b>   |
| $c = v_i \cdot \lambda_d$                 | <b>Always valid</b>             |
| $\lambda_d = \frac{1}{n} \cdot \lambda_i$ | <b>From the equations above</b> |

- A bit more involved is another basic relations coming from the Maxwell equations. It is the equation linking  $c$ , the speed of light in a material to the material "constants"  $\epsilon_r$  and the corresponding magnetic permeability  $\mu_0$  of vacuum and  $\mu_r$  of the material via

$$c = \frac{1}{(\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r)^{1/2}}$$

- Since most optical materials are not magnetic, i.e.  $\mu_r = 1$ , we obtain for the index of refraction of a dielectric material our relation [from above](#).

$$n = \frac{c_0}{c} = \frac{(\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r)^{1/2}}{(\mu_0 \cdot \epsilon_0)^{1/2}} = \epsilon_r^{1/2}$$

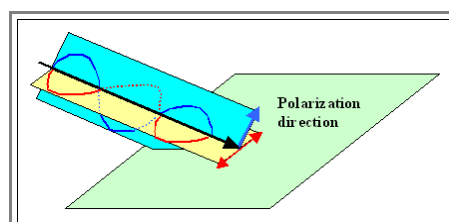
- Consult the [basic optics](#) module if you have problems so far.

If we now look at not-so-basic optics, we encounter the **Fresnel laws** of diffraction.

- Essentially, the **Fresnel laws** give the *intensity of the reflected beam* as a function of the *angle of incidence*, the *polarization* of the incident beam, and the *index of refraction* of the material.
- The Fresnel laws are not particularly easy to obtain (consult the basic module [Fresnel laws](#)), but the results are easy.

First, we must distinguish between the two basic polarization cases possible:

- The incident light might be polarized in such a way that the vector of the electrical field  $\underline{E}$  lies either *in* the plane of the material, or *perpendicular* to it, as shown below. Anything in between than can be decomposed into the two basic cases.



- Lets call the amplitudes of the reflected beam  $A_{para}$  for the case of the polarization being *parallel* to the plane (= surface of the dielectric), and  $A_{perp}$  for the case of the polarization being *perpendicular* to the plane (blue case) as shown above. For a unit amplitude of the incident beam, the Fresnel laws then state

$$A_{\text{para}} = - \frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} \quad A_{\text{perp}} = - \frac{\tan(\alpha - \beta)}{\tan(\alpha + \beta)}$$

▶ We can substitute the angle  $\beta$  by using the relation [from above](#) and the resulting equations then give the intensity of the reflected light as a function of the material parameter  $n$ . Possible, but the resulting equations are no longer simple.

- In order to stay simple and focus on the essentials, we will now consider only cases with about *perpendicular* incidence, i.e.  $\alpha \approx 0^\circ$ . This makes everything much easier.
- At *small* angles we may substitute the argument of the **sin** or **tan** for the full function, and obtain for *both* polarizations

$$A \approx - \frac{(\alpha - \beta)}{(\alpha + \beta)}$$

- Using the expression for  $n$  from above for small angles too, we obtain

$$n = \frac{\sin \alpha}{\sin \beta} \approx \frac{\alpha}{\beta}$$

- Now we keep in mind that we are usually interested in [intensities](#), and not in amplitudes. Putting everything together, we obtain for the reflectivity  $R$ , defined as the ratio of the intensity  $I_r$  of the reflected beam to the intensity  $I_i$  of the incident beam for almost perpendicular incidence

$$R = \frac{I_r}{I_i} = \frac{(n - 1)^2}{(n + 1)^2}$$

▶ The grand total of all of this is that if we know  $n$  and some basics about optics, we can answer most, *but not all* of the questions [from above](#). But so far we also did not use a *complex* index of refraction either.

- In essence, what is missing is any statement about the *attenuation* of the refracted beam, the **damping of the light** inside the dielectric - it is simply not contained in the equations presented so far.
- This cannot be right. Electromagnetic radiation does not penetrate arbitrarily thick (and still perfect) dielectrics - it gets pretty dark, for example, in deep water even if it is perfectly clear.
- In not answering the "damping" question, we even raise a new question: If we include damping in the consideration of wave propagation inside a dielectric, does it change the simple equations given above?

▶ The *bad* news is: It does! But relax: The *good* news is:

- All we have to do is to exchange the "simple" refractive index  $n$  by a *complex refractive index*  $n^*$  that is directly tied to the complex dielectric function, and everything is taken care of.
- We will see how this works in the next paragraph.