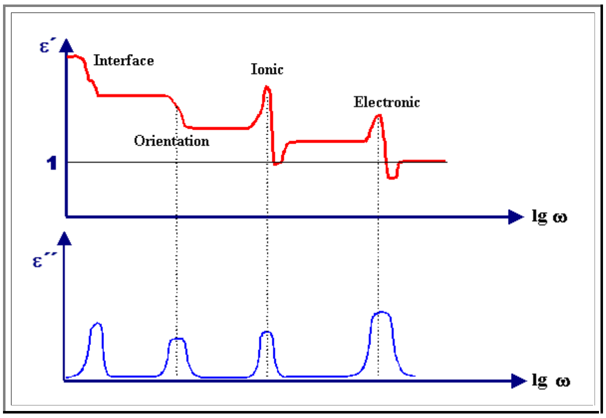
3.3.4 Complete Frequency Dependence of a Model Material

- The frequency dependence of a given material is superposition of the various mechanisms at work in this material. In the *idealized* case of a model material containing *all four basic mechanisms in their pure form* (a non-existent material in the real world), we would expect the following curve.



  - Note that ω is once more on a *logarithmic scale!*

- This is *highly idealized* - there is no material that comes even close! Still, there is a clear structure. Especially there seems to be a correlation between the real and imaginary part of the curve. That is indeed the case; *one* curve contains *all the information* about the other.
  - *Real* dielectric functions usually are only interesting for a small part of the spectrum. They may contain fine structures that reflect the fact that there may be *more than one* mechanism working at the same time, that the oscillating or relaxing particles may have to be treated by *quantum mechanical* methods, that the material is a *mix* of several components, and so on.
  - In the link a real dielectric function for a more complicated molecule is shown. While there is a lot of fine structure, the basic resonance function and the accompanying peak for **ε''** is still clearly visible.

- It is a general property of complex functions describing physical reality that under certain very general conditions, the real and imaginary part are directly related. The relation is called **Kramers-Kronig relation**; it is a *mathematical*, not a *physical* property, that only demands two very general conditions to be met:
  - Since two functions with a time or frequency dependence are to be correlated, one of the requirements is *causality*, the other one *linearity*.
  - The Kramers-Kronig relation can be most easily thought of as a *transformation* from one function to another by a black box; the functions being inputs and outputs. *Causality* means that there is no output before an input; *linearity* means that twice the input produces twice the output. Otherwise, the transformation can be anything.

- The Kramers-Kronig relation can be written as follows: For any complex function, e.g. $\epsilon(\omega) = \epsilon'(\omega) + i\epsilon''(\omega)$, we have the relations

$$\epsilon'(\omega) = \frac{-2\omega}{\pi} \int_0^\infty \frac{\omega^* \cdot \epsilon''(\omega^*)}{\omega^{*2} - \omega^2} \cdot d\omega^*$$

$$\epsilon''(\omega) = \frac{2\omega}{\pi} \int_0^\infty \frac{\epsilon'(\omega^*)}{\omega^{*2} - \omega^2} \cdot d\omega^*$$

- The Kramers-Kronig relation can be very useful for experimental work. If you want to have the dielectric function of some materials, you only have to measure one component, the other one can be calculated.

Questionaire

Multiple Choice questions to all of 3.3