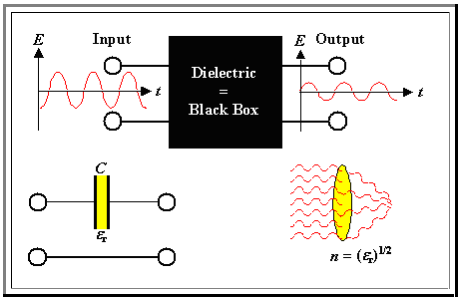
3.3 Frequency Dependence of the Dielectric Constant

3.3.1 General Remarks

- All polarization mechanisms respond to an electrical field by shifting masses around. This means that masses must be accelerated and de-accelerated, and this will always take some time. So we *must* expect that the (mechanical) response to a field will depend on the *frequency* ν of the electrical field; on how often per second it changes its sign.
  - If the frequency is very large, no mechanical system will be able to follow. We thus expect that at *very large frequencies all polarization mechanisms will "die out"*, i.e. there is no response to an extremely high frequency field. This means that the dielectric constant ϵ_r will approach **1** for $\nu \Rightarrow \infty$.
- It is best to consider our dielectric now as a "**black box**". A signal in the form of an alternating electrical field ***E*** goes in at the input, and something comes out at the output, as shown below. Besides the Black Box scheme, two possible real expressions of such an abstract system are shown: A parallel-plate capacitor containing a dielectric, and an optical lens with an index of refraction $n = \epsilon_r$. The input would be a simple alternating voltage in the capacitor case, and a light wave in the lens case.



- As long as our system is linear ("twice the input $\Rightarrow$ twice the output), a sinewave going in will produce a sinewave coming out, i.e. the frequency does not change.
  - If a sinewave goes in, the output then can only be a sinewave with an amplitude and a phase different from the input, as schematically shown above.
  - If a complicated signal goes in, we decompose it into its Fourier components, consider the output for all frequencies separately, and then do a Fourier synthesis.
- With complex notation, the input will be something like $E_{in} = E_{in} \cdot \exp(i\omega t)$; the output then will be $E_{out} = E_{out} \cdot \exp i(\omega t + \phi)$.
  - We just as well could write $E_{out} = f(\omega) \cdot E_{in}$ with $f(\omega) =$ complex number for a given ω or complex function of ω.
  - $f(\omega)$ is what we are after. We call this function that relates the output of a dielectric material to its input the **dielectric function** of the material. As we will see, the dielectric function is a well-defined and very powerful entity for any material - even if we cannot calculate it from scratch. We can however, calculate dielectric functions for some model materials, and that will give us a very good idea of what it is all about.
- Since the **index of refraction** ***n*** is directly given by $\epsilon_r^{1/2}$ (assuming that the material has no magnetic properties), we have a *first very general statement*:
  - There exist no microscopes with "optical" lenses for very high frequencies of electrical fields, which means electromagnetic radiation in the *deep ultraviolet* or soft **X-rays**. And indeed, there are no **X-ray microscopes** with lenses[1)] (however, we still have mirrors!) because there are no materials with $\epsilon_r > 1$ for the frequencies of **X**-rays.
- Looking at the polarization mechanisms discussed, we see that there is a fundamental difference in the *dynamics* of the mechanisms with regard to the response to changing forces:
  - In *two* cases (*electron and ionic polarization*), the electrical field will try to change the distance between the charges involved. In response, there is a restoring force that is (in our approximation) directly proportional to the separation distance of the dipole charges. We have, in mechanical terms, an *oscillator*.
  - The characteristic property of *any* such oscillating system is the phenomena of **resonance** at a specific frequency.
  - In the case of the *orientation polarization*, there is no direct mechanical force that "pulls" the dipoles back to random orientation. Instead we have many statistical events, that respond in their *average results* to the driving forces of electrical fields.
  - In other words, if a driving force is present, there is an equilibrium state with an (average) net dipole moment. If the driving force were to disappear suddenly, the ensemble of dipoles will assume a new equilibrium state (random distribution of the dipoles) within some characteristic time called **relaxation time**. The process knows no resonance phenomena, it is characterized by its **relaxation time** instead of a resonance frequency.
- We thus have to consider just the two basic situations: **Dipole relaxation** and **dipole resonance**. Every specific mechanism in real materials will fit one of the two cases.

[1)] Well, never say never. Lenses for X-rays *do* exist for a few years by now. However, if you would see the contraption, you most likely wouldn't recognize it as a *lens*. If you want to know more, turn to the resarch of Prof. Lengeler and his group: http://2b.physik.rtwh-aachen.de