2.5 Summary: Conductors

What counts are the *specific* quantities:

- Conductivity σ (or the specific resistivity $\rho = 1/\sigma$
- current density $\underline{j}$
- (Electrical) field strength · $\underline{E}$

$[\sigma] = (\Omega m)^{-1}$ = S/m;
S = 1/ Ω = "Siemens"
$[\rho] = \Omega m$

The basic equation for σ is:
n = concentration of carriers
μ = mobility of carriers

$$\sigma = |q| \cdot n \cdot \mu$$

Ohm's law states:
It is valid for metals, but not for all materials

$$\underline{j} = \sigma \cdot \underline{E}$$

σ (of conductors / metals) obeys (more or less) several rules; all understandable by looking at n and particularly μ.

Matthiesen rule
Reason: Scattering of electrons at defects (including phonons) decreases μ.

$$\rho = \rho_{Lattice}(T) + \rho_{defect}(N)$$

"$\rho(T)$ rule":
about **0,04 %** increase in resistivity per **K**
Reason: Scattering of electrons at phonons decreases μ

$$\Delta\rho = \alpha_\rho \cdot \rho \cdot \Delta T \approx \frac{0{,}4\%}{^{o}C}$$

Nordheim's rule:
Reason: Scattering of electrons at **B** atoms decreases μ

$$\rho \approx \rho_A + \text{const.} \cdot [B]$$

Major consequence: You can't beat the conductivity of pure **Ag** by "tricks" like alloying or by using other materials.
(Not considering superconductors).

Non-metallic conductors are *extremely* important.

Transparent conductors (TCO's)
("ITO", typically oxides)

No flat panels displays = no notebooks etc. without **ITO**!

Ionic conductors (liquid and solid)

Batteries, fuel cells, sensors, ...

Conductors for high temperature applications; corrosive environments, ..
(Graphite, Silicides, Nitrides, ...)

Example: **$MoSi_2$** for heating elements in corrosive environments (dishwasher!).

Organic conductors (and semiconductors)

The future High-Tech key materials?

Numbers to know (order of magnitude accuracy sufficient)

ρ(decent metals) about **2** μ**Ωcm**.
ρ(technical semiconductors) around **1** **Ωcm**.
ρ(insulators) > 1 GΩcm.

- No electrical engineering without conductors! Hundreds of specialized metal alloys exist just for "wires" because besides σ, other demands must be met, too:

> Money, Chemistry (try **Na**!), Mechanical and Thermal properties, Compatibility with other materials, Compatibility with production technologies, ...

  - Example for unexpected conductors being "best" compromise:

> Poly Si, Silicides, **TiN**, **W** in integrated circuits

- Don't forget Special Applications:

> Contacts (switches, plugs, ...); Resistors; Heating elements; ...

- Thermionic emission provides electron beams. The electron beam current (density) is given by the *Richardson equation*:
  - A_{theo} **= 120 A · cm^{-2} · K^{-2}** for free electron gas model A_{exp} ≈ **(20 - 160) A · cm^{-2} · K^{-2}**
  - E_A **=** work function ≈ **(2 - >6) eV**
  - Materials of choice: **W**, **LaB_6** single crystal

$$j = A \cdot T^2 \cdot \exp - \frac{E_A}{kT}$$

- High field effects (tunneling, barrier lowering) allow large currents at low ***T*** from small (nm) size emitter

> Needs **UHV**!

- There are several thermoelectric effects for metal junctions; always encountered in non-equilibrium.
  - *Seebeck effect:* Thermovoltage develops if a metal A-metal B junction is at a temperature different form the "rest", i.e. if there is a temperature gradeient

> Essential for measuring (high) temperatures with a "thermoelement" Future use for efficient conversion of heat to electricity ???

  - *Peltier effect:* Electrical current *I* through a metal - metal (or metal - semiconductor) junction induces a temperature gradient ∝ *I*, i.e. one of the junction may "cool down".

> Used for electrical cooling of (relatively small) devices. Only big effect if electrical heating (∝ I^2) is small.

- Electrical current can conducted by *ions* in
  - Liquid electrolytes (like H_2SO_4 in your "lead - acid" car battery); including gels
  - Solid electrolytes (= ion-conducting crystals). Mandatory for fuel cells and sensors
  - Ion beams. Used in (expensive) machinery for "nanoprocessing".

> **Challenge**: Find / design a material with a "good" ion conductivity at room temperature

- Basic principle
  - *Diffusion current* j_{diff} driven by concentration gradients **grad(*c*)** of the charged particles (= ions here) equilibrates with the
  - *Field current* j_{field} caused by the internal field always associated to concentration gradients of charged particles plus the field coming from the outside

$$j_{diff} = -D \cdot \text{grad}(c)$$

$$j_{field} = \sigma \cdot E = q \cdot c \cdot \mu \cdot E$$

- Diffusion coefficient ***D*** and mobility **μ** are linked via theEinstein relation;
  concentration ***c*(*x*)** and potential ***U*(*x*)** or field ***E*(*x*) = –d*U*/d*x***by the Poisson equation.

$$\mu = eD/kT$$

$$-\frac{d^2U}{dx^2} = \frac{dE}{dx} = \frac{e \cdot c(x)}{\epsilon\epsilon_0}$$

Immediate results of the equations from above are:

- In equilibrium we find a preserved quantity, i.e. a quantity independent of ***x*** - the electrochemical potential V_{ec}:

$$V_{ec} = \text{const.} = e \cdot U(x) + kT \cdot \ln c(x)$$

- If you rewrite the equaiton for ***c*(*x*)**, it simply asserts that the particles are distributed on the energy scale according to the Boltzmann distrubution:

$$c(x) = \exp - \frac{(Vx) - V_{ec}}{kT}$$

- Electrical field *gradients* and concentration *gradients* at "contacts" are coupled and non-zero on a length scale given by the **Debye length** d_{Debye}
- The Debye length is an extremely important material parameter in "**ionics**" (akin to the space charge region width in semiconductors); it depends on temperature ***T*** and in particular on the (bulk) concentration c_0 of the (ionic) carriers.
- The Debye length is not an important material parameter in metals since it is so small that it doesn't matter much.

$$d_{Debye} = \left(\frac{\epsilon \cdot \epsilon_0 \cdot kT}{e^2 \cdot c_0}\right)^{1/2}$$

The potential difference between two materials (her ionic conductors) in close contact thus...

- ... extends over a length given (approximately) by :

$$d_{Debye}(1) + d_{Debye}(2)$$

- ... is directly given by the Boltzmann distribution written for the energy:
  (with the c_i =equilibrium conc. far away from the contact.

$$\frac{c_1}{c_2} = \exp - \frac{e \cdot \Delta U}{kT} \quad \textbf{Boltzmann}$$

- The famous *Nernst equation*, fundamental to ionics, is thus just the Boltzmann distribution in disguise!

$$\Delta U = - \frac{kT}{e} \cdot \ln \frac{c_1}{c_2} \quad \textbf{Nernst's equation}$$

"Ionic" sensors (most famous the $\mathbf{ZrO_2}$ - based $\mathbf{O_2}$ sensor in your car exhaust system) produce a voltage according to the Nernst equation because the concentration of ions on the exposed side depends somehow on the concentration of the species to be measured.

[Questionaire](#)
All multiple choice questions zu 2. Conductors