Solution to Exercise 1.3-2

Derive numbers for v_0, v_D, τ, and l

First Task: Derive a number for $\mathbf{v_0}$ (at room temperature). We have

$$v_0 = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{8{,}6 \cdot 10^{-5} \cdot 300}{9{,}1 \cdot 10^{-31}} \, \frac{\mathrm{eV} \cdot \mathrm{K}}{\mathrm{K} \cdot \mathrm{kg}}\right)^{1/2} = 1{,}68 \cdot 10^{14} \cdot \left(\frac{\mathrm{eV}}{\mathrm{kg}}\right)^{1/2}$$

- The dimension "square root of **eV/kg**" does not look so good - for a velocity we would like to have **m/s**. In looking at the energies we equated kinetic energy with the classical dimension **[kg · m^2/s^2] = [J]** with thermal energy **k*T*** expressed in **[eV]**. So let's convert **eV** to **J** (use the link) and see if that solves the problem. We have **1 eV = 1,6 · 10^{-19} J = 1,6 · 10^{-19} kg · m^2 · s^{-2}** which gives us

$$v_0 = 1{,}68 \cdot 10^{14} \cdot \left(\frac{1{,}6 \cdot 10^{-19}\ \mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{kg} \cdot \mathrm{s}^2}\right)^{1/2} = 5{,}31 \cdot 10^4\ \mathrm{m/s} = 1{,}91 \cdot 10^5\ \mathrm{km/hr}$$

Possibly a bit surprising - those electrons are no sluggards but move around rather fast. Anyway, we have shown that a value of ≈ **10^4 m/s** as postulated in the backbone is really OK.

- Of course, for $T \to 0$, we would have $v_0 \to 0$ - which should worry us a bit ???? If instead of room temperature (***T* = 300 K**) we would go to let's say **1200 K** , we would just double the average speed of the electrons.

Second Task: Derive a number for τ. We have

$$\tau = \frac{\sigma \cdot m}{n \cdot e^2}$$

First we need some number for the concentration of free electrons per **m^3**. For that we complete the table given, noting that for the number of atoms per **m^3** we have to divide the density by the atomic weight.

Atom	Density [kg · m^{-3}]	Atomic weight × 1,66 · 10^{-27} kg	Conductivity σ × 10^5 [Ω^{-1} · m^{-1}]	No. Atoms [m^{-3}] × 10^{28}
Na	970	23	2,4	2,54
Cu	8.920	64	5,9	8,40
Au	19.300	197	4,5	5,90

- So let's take **5 · 10^{28} m^{-3}** as a good order of magnitude guess for the number of atoms in a **m^3**, and for a first estimate some average value **σ = 5 · 10^5 [Ω^{-1} · m^{-1}]**. We obtain

$$\tau = \frac{5 \cdot 10^5 \cdot 9{,}1 \cdot 10^{-31}}{5 \cdot 10^{28} \cdot (1{,}6 \cdot 10^{-19})^2} \, \frac{\mathrm{kg} \cdot \mathrm{m}^3}{\Omega \cdot \mathrm{m} \cdot \mathrm{A}^2 \cdot \mathrm{s}^2} = 3{,}55 \cdot 10^{-16} \, \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{V} \cdot \mathrm{A} \cdot \mathrm{s}^2}$$

Well, somehow the whole thing would look much better with the unit **[s]**. So let's see if we can remedy the situation.

- Easy: Volts times Amperes equals *Watts* which is power, e.g. energy per time, with the unit **[J · s^{-1}] = kg · m^2 · s^{-3}**. Insertion yields

$$\tau = 1{,}42 \cdot 10^{-28} \frac{\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^3}{\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^2} = 3{,}55 \cdot 10^{-16}\ \mathrm{s} = 0.35\ \mathrm{fs}$$

- The backbone thus is right again. The scattering time is in the order of femtosecond which is a short time indeed. Since all variables enter the equation linearly, looking at somewhat other carra ier densities (e.g. more than **1** electron per atom) or conductivities does not really change the general picture very much.

- Third Task: Derive a number for $\mathbf{v_D}$. We have (for a field strength $\boldsymbol{E}$ **= 100 V/m = 1 V/cm**)

$$|v_D| = \frac{E \cdot e \cdot \tau}{m} = \frac{100 \cdot 1{,}6 \cdot 10^{-19} \cdot 3{,}55 \cdot 10^{-16}}{9{,}1 \cdot 10^{-31}} \frac{\mathrm{V} \cdot \mathrm{C} \cdot \mathrm{s}}{\mathrm{m} \cdot \mathrm{kg}} = 6{,}24 \cdot 10^{-3} \frac{\mathrm{V} \cdot \mathrm{A} \cdot \mathrm{s}^2}{\mathrm{m} \cdot \mathrm{kg}}$$

$$= 6{,}24 \cdot 10^{-3} \frac{\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^2}{\mathrm{m} \cdot \mathrm{kg} \cdot \mathrm{s}^3} = 6{,}24 \cdot 10^{-3}\ \mathrm{m/s} = 6{,}24\ \mathrm{mm/s}$$

- This is somewhat larger than the value given in the backbone text.
  - However - a field strength of **1 V/cm** applied to a *metal* is huge! Think about the current density $\boldsymbol{j}$ you would get if you apply **1 V** to a piece of metal **1 cm** thick.
  - It is actually $j = \sigma \cdot E = 5 \cdot 10^7\ [\Omega^{-1} \cdot \mathrm{m}^{-1}] \cdot 100\ \mathrm{V/m} = 5 \cdot 10^9\ \mathrm{A/m^2} = 5 \cdot 10^5\ \mathrm{A/cm^2}$!
  - For a more "reasonable" current density of $10^3\ \mathrm{A/cm^2}$ we have to reduce $\boldsymbol{E}$ hundredfold and then end up with $|v_D| = 0{,}0624\ \mathrm{mm/s}$ - and that is slow indeed!

- Fourth Task: Derive a number for $\boldsymbol{l}$. We have

$$l_{min} = 2 \cdot v_0 \cdot \tau = 2 \cdot 5{,}31 \cdot 10^4 \cdot 3{,}55 \cdot 10^{-16}\ \mathrm{m} = 3{,}77 \cdot 10^7\ \mathrm{m} = 0{,}0377\ \mathrm{nm}$$

  - Right again! If we add the comparatively miniscule $\mathbf{v_D}$, nothing would change. Decreasing the temperature would lower $\boldsymbol{l}$ to eventually zero, or more precisely, to $2 \cdot v_D \cdot \tau$ and thus to a value far smaller than an atom..