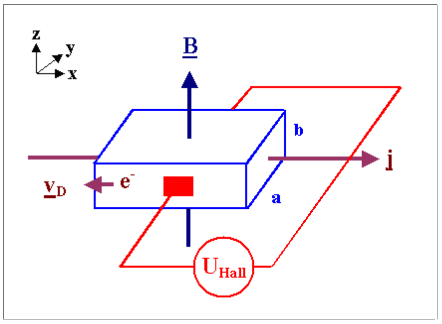
This Module comes from "Introduction to Materials Science II"; where you will find the most recent version. The link takes you there.

Required Reading

1.3.4 The Hall Effect

- This subchapter introduces *two* important topics: The **Hall effect** as an important observation in materials science and at the same time another irrefutable proof that classical physics just can't hack it when it comes to electrons in crystals.
  - The Hall effect describes what happens to current flowing through a conducting material - a metal, a semiconductor - if it is exposed to a magnetic field $\underline{B}$.
  - We will look at this in *classical* terms; again we will encounter a fundamental problem.
- The standard geometry for doing an experiment in its most simple form is as follows:



Relevant vectors; the current is carried by electrons

  - A magnetic field $\underline{B}$ is employed perpendicular to the current direction $\underline{j}$, as a consequence a *potential difference* (i.e. a *voltage*) develops at right angles to both vectors.
  - In other words: A **Hall voltage** U_{Hall} will be measured perpendicular to $\underline{B}$ and $\underline{j}$.
  - In yet other words: An electrical field $\underline{E}_{Hall}$ develops in ***y***-direction
  - That is already the essence of the Hall effect.

- It is relatively easy to calculate the magnitude of the *Hall voltage* U_{Hall} that is induced by the magnetic field $\underline{B}$.
  - First we note that we must also have an electrical field $\underline{E}$ parallel to $\underline{j}$ because it is the driving force for the current.
  - Second, we know that a magnetic field at right angles to a current causes a force on the moving carriers, the so-called **Lorentz force** F_L, that is given by

$$\underline{F}_L = q \cdot (\underline{v}_D \times \underline{B})$$

  - We have to take the drift velocity $\underline{v}_D$ of the carriers, because the other velocities (and the forces caused by these components) cancel to zero on average. The vector product assures that $\underline{F}_L$ is perpendicular to $\underline{v}_D$ and $\underline{B}$.
  - Note that instead the usual word "electron" the neutral term *carrier* is used, because in principle an electrical current could also be carried by charged particles other then electrons, e.g. positively charged ions. Remember a simple but important picture given before!
- For the geometry above, the Lorentz force $\underline{F}_L$ has only a component in ***y*** - direction and we can use a scalar equation for it. F_y is given by

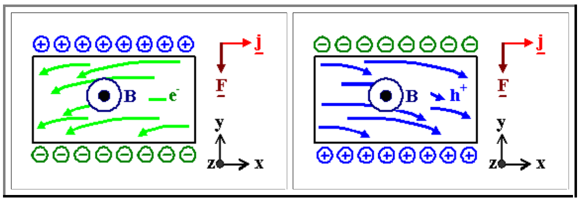
$$F_y = -q \cdot v_D \cdot B_z$$

  - We have to be a bit careful, however: We know that the force is in ***y***-direction, but we do longer know the sign. It changes if either q, v_D, or B_z changes direction and we have to be aware of that.
- With $\underline{v}_D = \mu \cdot \underline{E}$ and μ = mobility of the carriers, we obtain a rather simple equation for the force

$$F_y = -q \cdot \mu \cdot E_x \cdot B_z$$

  - It is important to note that for a fixed current density j_x the direction of the Lorentz force is independent of the sign of the charge carriers (the sign of the charge and the sign of the drift velocity just cancel each other).

- This means that the current of carriers will be deflected from a straight line in ***y***-direction. In other words, there is a component of the velocity in ***y***-direction and the surfaces perpendicular to the ***y***-direction will become charged as soon as the current (or the magnetic field) is switched on. The flow-lines of the carriers will look like this:



  - The charging of the surfaces is unavoidable, because some of the carriers eventually will end up at the surface where they are "stuck".
  - Notice that the sign of the charge for a given surface depends on the sign of the charge of the carriers. Negatively charged electrons (**e**$^-$ in the picture) end up on the surface opposite to positively charged carriers (called **h**$^+$ in the picture).
  - Notice, too, that the direction of the force $\underline{F}_y$ is the same for both types of carriers, simply because both q and $\underline{v}_D$ change signs in the force formula
- The surface charge then induces an electrical field E_y in ***y***-direction which opposes the Lorentz force; it tries to move the carriers back.
  - In *equilibrium*, the Lorentz force $\underline{F}_y$ and the force from the electrical field E_y in ***y***-direction (which is of course simply $q \cdot E_y$) must be equal with opposite signs. We obtain

$$q \cdot E_y = -q \cdot \mu \cdot E_x \cdot B_z$$

$$E_y = -\mu \cdot E_x \cdot B_z$$

- The Hall voltage U_{Hall} now is simply the field in ***y***-direction multiplied by the dimension d_y in ***y***-direction.
  - It is clear then that the (easily measured) Hall voltage is a *direct measure* of the mobility **μ** of the carriers involved, and that its **sign** or polarity will change if the sign of the charges flowing changes.
- It is customary to define a **Hall coefficient** R_{Hall} for a given material.
  - This can be done in different, but equivalent ways. In the link we look at a definition that is particularly suited for measurements. Here we use the following definition:

$$R_{Hall} = \frac{E_y}{B_z \cdot j_x}$$

- In other words, we expect that the Hall voltage $E_y \cdot d_y$ (with d_y = dimension in ***y***-direction) is proportional to the current(density) j and the magnetic field strength B, which are, after all, the main experimental parameters (besides the trivial dimensions of the specimen):

$$E_y = R_{Hall} \cdot B_z \cdot j_x$$

- The Hall coefficient is a material parameter, indeed, because we will get different numbers for R_{Hall} if we do experiments with identical magnetic fields and current densities, but different materials. The Hall coefficient, as mentioned before, has interesting properties:
  - R_{Hall} will change its sign, if the sign of the carriers is changed because then E_y changes its sign, too. It thus indicates in the most unambiguous way imaginable if positive or negative charges carry the current.
  - R_{Hall} allows to obtain the mobility **μ** of the carriers, too, as we will see immediately
- R_{Hall} is easily calculated: Using the equation for E_y from above, and the basic equation $j_x = \sigma \cdot E_x$, we obtain for *negatively* charged carriers:

$$R_{Hall} = -\frac{\mu \cdot E_x \cdot B_z}{\sigma \cdot E_x \cdot B_z} = -\frac{\mu}{\sigma}$$

- Measurements of the Hall coefficient of materials with a *known* conductivity thus give us *directly* the mobility of the carriers responsible for the conductance.

- The **–** sign above is obtained for *electrons*, i.e. *negative* charges.
- If positively charged carriers would be involved, the Hall constant would be positive.
- Note that while it is not always easy to measure the numerical value of the Hall voltage and thus of ***R*** with good precision, it is the easiest thing in the world to measure the *polarity* of a voltage.

Lets look at a few experimental data:

Material	**Li**	**Cu**	**Ag**	**Au**	**Al**	**Be**	**In**	Semiconductors (e.g. **Si**, G**e**, **GaAs**, **InP**,...)
R (× 10^{-24}) cgs units	–1,89	–0,6	–1,0	–0,8	+1,136	+2,7	+1,774	*positive* or *negative* values, depending on "doping"

Comments:
1. the *positive* values for the metals were measured under somewhat special conditions (low temperatures; single crystals with special orientations), for other conditions negative values can be obtained, too.
2. The units are not important in the case, but multiplying with **$9 \cdot 10^{13}$** yields the value in **m^3/Coulomb**

Whichever way we look at this, one conclusion is unavoidable:

- In certain materials including *metals*, the particles carrying the electrical current are *positively charged* under certain conditions. And this is *positively not possible* in a classical model that knows only *negatively charged electrons* as carriers of electrical current in solids!

Again we are forced to conclude:

There is no way to describe conductivity in metals and semiconductors with *classical* physics!

Questionaire

Multiple Choice Fragen zu 1.3.4