Solution to Exercise 2.4.1

Illustration

1. How many electrons per **cm²** do you need on the surface a capacitor made of parallel metal plates in air with an area of **1 cm²** and distance **1 cm** to provide for some field ***E*** ending there?

The relation between the field ***E*** resulting from a homogeneous two-dimensional charge distribution and the charge density ρ_{area} is

$$E = \frac{Q}{\epsilon_0 \cdot A} = \frac{\rho_{area}}{\epsilon_0}$$

- with ***Q*** = charge in **[C]**, ***A*** = area considered, ρ_{area} = areal density of the charge.
- Compare the two formulas for the capacity ***C*** of the capacitor formed by the parallel plates if you are not sure about the equation above. We have:

$$C = \frac{\epsilon_0 \cdot A}{d} = \frac{Q}{U} \quad \text{and} \quad E = \frac{U}{d}$$

2. What would be the maximum density for reasonable field strengths up to an ultimate limit of about **10 MV/cm**? (For higher field strengths you will get violent discharge).

Lets look at some numbers (ϵ_0 **= 8,854 · 10^{-14} C/Vcm**)

Field strength	**10^3 V/cm** (rather low)	**10^5 V/cm** (breakdown limit of "normal dielectrics"	**10^7 V/cm** (close to ultimate breakdown limit)
ρ_{area}	**8,85 · 10^{-11} C/cm²**	**8,85 · 10^{-9} C/cm²**	**8,85 · 10^{-7} C/cm²**

3. How does this number compare to the average volume density of electron in metals. Consider, e.g., from how far away from the surface you have to collect electrons to achieve the required surface density, if you allow the volume density in the afflicted volume to decrease by ***x*** **%**?

The average volume density of electrons in metals is about **1** electron/atomic volume.

- Lets keep thing easy and take for the size d_{atom} **= 1 Å**, which gives **1 Å³ = 10^{-3} nm³** for the volume of one atom. The *volume density* of atoms or electrons per **cm³** is thus ρ_{volume} **= 10^{24} electrons/cm³**.
- The *areal density* is whatever is contained in a volume with an extension of just one atom diameter in one direction, i.e.

$$\rho_{areal} = \rho_{volume} \cdot d_{atom} = 10^{-17} \text{ electrons/cm}^2$$

- If we want to collect a surplus charge of $Q_{surplus}$ **= 8,85 · 10^{-7} C/cm²**, the maximum charge from above, from a volume $V_{surplus}$ by reducing the concentration of **10^{-24} electrons/cm³** by ***x*** **%**, we have

$$Q_{surplus} = \frac{\rho_{volume} \cdot d}{100 \cdot x}$$

$$d = \frac{100 \cdot x \cdot Q_{surplus}}{\rho_{volume}} = \frac{x \cdot 8{,}85 \cdot 10^{-5}}{10^{24}} \text{ cm} = x \cdot 8{,}85 \cdot 10^{-29} \text{ cm}$$

In words: Whatever value we like for **x**, we only have to change the volume concentration of the electrons in an extremely thin layer a tiny little bit to produce *any* areal charge densities needed - in *metals*, that is!