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Exercises "Electronic Materials"

#6

Exercise 6: Resonant system

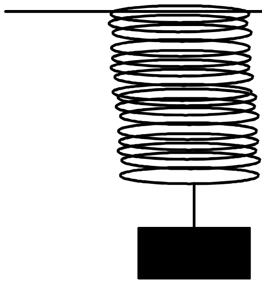

Fig. 1: A mass *m* hanging on a spring with a spring constant k_S.

A mass *m* on a spring (with a spring constant k_S) is taken as a model system to describe systems which have a linear backdriving force in general:

$$m\frac{d^2x}{dt^2} = -k_S x\,. \qquad \textbf{(1)}$$

a) Discuss shortly the parts of **equation (1)**.
b) Show that $x(t) = x_0 \exp\{i\omega_0 t + i\varphi\}$ with $\omega_0 = \sqrt{k_S / m}$ is a solution of equation **(1)**. What is the meaning of a phase $\varphi \neq 0$? What is the meaning of a resonance frequency ω_0?
c) Transfer this picture to describe electronic polarization. What is the mass *m* and the spring constant k_S in this case?

In most cases the friction force is proportional to the velocity of the mass *m* and can thus be directly introduced into **(1)**:

$$m\frac{d^2x}{dt^2} = -k_S x - mk_F\frac{dx}{dt}\,. \qquad \textbf{(2)}$$

d) Explain the minus sign of the friction force. What could be friction in the case of electronic polarization?
e) Show that the resonance frequency shifts to $\omega_1 = \frac{ik_F}{2} + \sqrt{{\omega_0}^2 - (k_F/2)^2}$ for **equation (2)**.

Finally a periodic exicitation $F_0 \exp\{i\omega t\}$ is added to the system. The whole movement of the mass m is given by

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -mk_\mathrm{F}\frac{\mathrm{d}x}{\mathrm{d}t} - k_\mathrm{S}x + F_0\exp\{i\omega t\}. \qquad \textbf{(3)}$$

f) Which is the angular frequency of the oscillating mass in steady state?
g) Show that

$$x(\omega,t) = x(\omega)\exp\{i\omega t\} = \left[\frac{F_0}{m}\left(\frac{{\omega_0}^2 - \omega^2}{({\omega_0}^2 - \omega^2)^2 + (k_\mathrm{F}\omega)^2} - i\frac{k_\mathrm{F}\omega}{({\omega_0}^2 - \omega^2)^2 + (k_\mathrm{F}\omega)^2}\right)\right]\cdot\exp\{i\omega t\}$$

is a solution for **equation (3)**.

Hint: $\dfrac{{\omega_0}^2 - \omega^2}{({\omega_0}^2 - \omega^2)^2 + (k_\mathrm{F}\omega)^2} - i\dfrac{k_\mathrm{F}\omega}{({\omega_0}^2 - \omega^2)^2 + (k_\mathrm{F}\omega)^2} = \dfrac{1}{{\omega_0}^2 - \omega^2 + ik_\mathrm{F}\omega}$.

h) Draw qualitatively $x(t)$ for the cases **(1), (2)** and **(3)**. Draw the imaginary and real part of $x(\omega)$ for case **(3)**.
i) What is the external "driving force" F_0 in the case of an electromagnetic wave?
j) If the mass m is enlarged what happens to the resonance frequency, the imaginary and real part of $x(\omega)$?
k) If the friction constant k_F gets smaller, what will be the consequences?
l) What ingredient is missing in the case of orientation polarization so that you can't handle this case by a resonance analysis? How does the frequency dependence (real and imaginary part) for a relaxation phenomenon look like?