Solution to Exercise 1.3-1

Derive and Discuss numbers for μ

- First Task: Derive numbers for the mobility **μ**.
  - First we need typical conductivities and electron densities in *metals*, which we can take from the table in the link.
  - At the same time we expand the table a bit

Material	ρ [Ω cm]	σ [Ω^{-1} cm^{-1}]	**Density** $d \times 10^3$ [kg m^{-3}]	**Atomic weight** w [× 1u = 1,66 · 10^{-27} kg]	$n = d/w$ [m^{-3}]
Silver Ag	$1{,}6 \cdot 10^{-6}$	$6.2 \cdot 10^5$	10,49	107,9	$5{,}85 \cdot 10^{28}$
Copper Cu	$1{,}7 \cdot 10^{-6}$	$5.9 \cdot 10^5$	8,92	63,5	$8{,}46 \cdot 10^{28}$
Lead Pb	$21 \cdot 10^{-6}$	$4.8 \cdot 10^4$	11,34	207,2	$3{,}3 \cdot 10^{28}$

- For the mobility **μ** we have the equation

$$\mu = \frac{\sigma}{q \cdot n}$$

- With q = elementary charge **= 1,60 10^{-19} C** we obtain, for example for μ_{Ag}

$$\mu_{Ag} = \frac{6{,}2 \cdot 10^5}{1{,}6 \cdot 10^{-19} \cdot 5{,}85 \cdot 10^{28}} \; \frac{m^3}{C \cdot \Omega \cdot cm} = 66{,}2 \; \frac{cm^2}{C \cdot \Omega}$$

- The unit is a bit strange, but rembering that **[C] = [A · s]** and **[Ω] = [V/A]**, we obtain

$$\mu_{Ag} = 66{,}2 \; \frac{cm^2}{Vs}$$

$$\mu_{Cu} = 43.6 \; \frac{cm^2}{Vs}$$

$$\mu_{Pb} = 9{,}1 \; \frac{cm^2}{Vs}$$

Second Task: Derive numbers for the drift velocity v_D by considering a reasonable field strength.

- The mobility **μ** was defined as

$$\mu = \frac{v_D}{E}$$

or

$$v_D = \mu \cdot E$$

So what is a reasonable field strength in a metal?

- Easy. Consider a cube with side lengt l **= 1 cm**. Its resistance R is given by

$$R = \frac{\rho \cdot l}{F} = \rho\ \Omega$$

- A **Cu** or **Ag** cube thus would have a resistance of about $1{,}5 \cdot 10^{-6}\ \Omega$**.** Applying a voltage of **1 V**, or equivalently a field strength of **1 V/cm** thus produces a current of $I = U/R \approx$ **650 000 A** or a current density j **= 650 000 A/cm**^{2}
- That seems to be an awfully large current. Yes, but it is the kind of current *density* encountered in integrated circuits! Think about it!
- Nevertheless, the wires in your house carry at most about **30 A** (above that the fuse blows) with a cross section of about **1 mm**^{2}; so a reasonable current density is **3000 A/cm**^{2}, which we will get for about $U = 1{,}5 \cdot 10^{-6}\ \Omega \cdot 3000\ \text{A} = 4{,}5\ \text{mV}$.
- For a rough estimate we then take a field strength of **5 mV/cm** and a mobility of **50 cm**^{2}**/Vs** and obtain

$$v_D = 50 \cdot 5\ \frac{\text{mV} \cdot \text{cm}^2}{\text{cm} \cdot \text{V} \cdot \text{s}} = 0{,}25\ \frac{\text{cm}}{\text{s}} = 2{,}5\ \frac{\text{mm}}{\text{s}}$$

That should come as some surprise! The electrons only have to move v e r y s l o w l y *on average* in the current direction (or rather, due to sign conventions, against it).

- Is that true, or did we make a mistake?
- It *is* true! However, it does *not* mean, that electrons will not run around like crazy inside the crystal, at very high speeds. It only means that their *net* movement in current anti-direction is very slow.
- Think of an single fly in a fly swarm. Even better read the module that discusses this analogy in detail. The flies are flying around at high speed like crazy - but the fly swarm is not going anywhere as long as it stays in place. There is then no drift velocity and no net fly current!