Solution to Exercise 4.1-1 "Lifetime of Positrons"

Illustration

- Show that the solution of the differential equations for the positron concentrations n_1 and n_2

$$\frac{dn_1}{dt} = -(\lambda_1 + \nu \cdot c_V) \cdot n_1$$

$$\frac{dn_2}{dt} = -\lambda_2 \cdot n_2 + \nu \cdot c_V \cdot n_1$$

- The way to obtain the desired equation shall only be sketched.
  - First, the coupled differential equation from above need to be solved for the initial condition

$$n_1(t = 0) + n_2(t = 0) = n_0$$

  - n_0 is the number of thermalized positrons in the crystal at the beginning of the experiment: i.e. at $t = 0$
- This is easy to do since the first differential equation does not contain n_2.
  - The solution must be

$$n_1(t) = A \cdot \exp-(\lambda_1 + \nu \cdot c_V)t$$

- Insertion in the second differential equation and using the initial condition yields

$$n(t) = n_0 \cdot \frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_2 + \nu \cdot c_V} \exp-(\lambda_1 + \nu \cdot c_V)t + n_0 \cdot \frac{\nu \cdot c_V}{\lambda_1 - \lambda_2 + \nu \cdot c_V} \exp-\lambda_2 t$$

- We have two components decaying with two lifetimes, τ_1 and τ_2, given by

$$\tau_1 = \frac{1}{\lambda_1 + \nu \cdot c_V}$$

$$\tau_2 = \frac{1}{\lambda_2}$$

- Measurements usually only yield an *average* lifetime $<\tau>$
  - The *average* lifetime is not simply the average of τ_1 and τ_2 because we need *weighted* averages, i.e.

$$<\tau>= \frac{1}{n_0} \int_0^\infty t \frac{dn(t)}{dt} dt$$

$$<\tau>= \tau_2 \cdot \frac{1 + \tau_2 \nu \cdot c_V}{1 + \tau_1 \nu \cdot c_V}$$

Doing the integral takes a few lines, but it is not too difficult - try it!