1.6 Representation of numbers: Binary and hexadecimal

For any counting system, each individual digit of a given number represents a value according to its position relative to the unit position[10] and according to the base underlying the respective system:

$$\begin{aligned} 147.35_{10} &= 1\times 10^2 + 4\times 10^1 + 7\times 10^0 + 3\times 10^{-1} + 5\times 10^{-2} && \text{— decimal system;} \\ 101.01_2 &= 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2} && \text{— binary system.} \end{aligned} \tag{1.1}$$

The conversion from integer decimal to binary numbers is done by repeated division by 2. The remainder of this division, which is either 0 or 1, successively provides the digits of the binary number (from right to left). As an example it is shown how to convert the decimal value 6 to the binary system:

	/2	**result**	**remainder**	**total**
6_{10}	/2	3	0	
3	/2	1	1	
1	/2	0	1	$\Rightarrow 110_2$

(1.2)

The conversion of fractional decimal to fractional binary numbers smaller than 1 is done by repeated multiplication by 2. The value at the decimal point, which is either 0 or 1, is the next binary digit (from left to right), and the remaining fraction is further multiplied by 2. This is shown as an example for the decimal value 3/8:

	×2	**result**	**at dec. point**	**remaining**	**total**
0.375_{10}	×2	0.75	0	.75	
0.75	×2	1.5	1	.5	
0.5	×2	1.0	1	.0	$\Rightarrow 0.011_2$

(1.3)

As in the decimal system, an integer exponent $n > 0$ tells the number of trailing zeros of 2^n (or, equivalently, the number of places of the next-smallest integer):

$$2^n = \underbrace{1000...0}_{n+1\text{ places}}\,, \quad 2^n - 1 = \underbrace{111...1}_{n\text{ places}}\,. \tag{1.4}$$

On the other hand, an integer exponent $n < 0$ tells the number of leading zeros of 2^n up to the unit position (or, equivalently, the number of nonzero places of $1 - 2^n$):

$$2^{-|n|} = \underbrace{0.00...01}_{|n|+1\text{ places}}\,, \quad 1 - 2^{-|n|} = 0.\underbrace{11...11}_{|n|\text{ places}}\,. \tag{1.5}$$

In the *hexadecimal system* (abbreviated as "hex"), the base 16 is used. Therefore, at each place of a number, values between zero and fifteen can occur. In this system, the letters A through F serve as signs to denote the values ten through fifteen. The hexadecimal digits therefore are: 0, 1, 2, ..., 9, A, B, C, D, E, F. Example:

$$17\mathrm{F}_{\mathrm{hex}} = (1\times 16^2 + 7\times 16^1 + 15\times 16^0)_{10} = 383_{10}\,. \tag{1.6}$$

[10] For integer numbers, the unit position is the last digit to the right; for fractional numbers it is the one left of the (respective) point.