## 2.2 Polynomial interpolation: Introduction

The most simple case of interpolation is a straight line between two points. (Extending this line beyond the two points is referred to as *extrapolation.*) Three points that do not lie on a common straight line need to be connected by a curve. Then, a unique solution for a connecting curve is provided by a parabola (having three parameters which can be determined using the three data points). Analogously, for four points which neither can be connected by a single straight line nor a single parabola, the unique connecting curve is a cubic function (polynomial of $3^{\text{rd}}$ degree having four parameters). In general terms:

**Theorem:** *interpolating polynomial* (without proof)
For $n+1$ given points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ with $x_i \neq x_j$ for $i \neq j$ there exists exactly one polynomial $p(x)$ of degree $\leq n$ with $p(x_i) = y_i$ for all $i \in \{0, 1, 2, \ldots, n\}$. We call $p(x)$ the interpolating polynomial for the given points.

Example:
Consider the following points: $(x_0, y_0) = (0, 1)$; $(x_1, y_1) = (1, 6)$; $(x_2, y_2) = (2, 15)$. They define a parabola, i.e. a quadratic polynomial $p(x) = a + bx + cx^2$. The coefficients are found from the three conditions $p(0) = 1, p(1) = 6, p(2) = 15$, which leads to the solution $p(x) = 1 + 3x + 2x^2$.