Solution to 5.2-3: Attenuation of Light

Illustration

$$E_x = \exp -\frac{\omega \cdot \kappa \cdot x}{c} \cdot \exp[\,i \cdot (k_x \cdot x - \omega \cdot t)]$$

Decreasing amplitude — **Plane wave**

Starting from the equation at right

Give maximal values for κ (damping constant, attenuation index, extinction coefficient) if a penetration depth of **1m**, **100 m** 10^4 **m** is specified for the light intensity.

The *intensity* I is proportional to E^2 and thus decreases with $I = I_0 \exp - \{(2\omega \cdot \kappa \cdot x)/c\}$.

- We have $\ln\{I/I_0\} = -(2\omega \cdot \kappa \cdot x)/c$, or $\kappa = -\{c/2x\omega\}\ln\{I/I_0\}$.
  If we assume that $I/I_0 = 1/e$ as a measure of still sufficient intensity, we have $\ln(I/I_0) = -1$ and thus obtain $\kappa = (1/x) \cdot (c/2\omega)$.
- Taking $\omega = 10^{16}\ s^{-1}$ we have $c/2\omega = 3 \cdot 10^8\ ms^{-1} / 2 \cdot 10^{16}\ s^{-1} = 1{,}5 \cdot 10^{-8}$ **m**. We arrive at the following table for $\kappa = 1{,}5 \cdot 10^{-8}/x$ for x given in meter (m).

x	κ
1 m	$1{,}5 \cdot 10^{-8}$
100 m	$1{,}5 \cdot 10^{-10}$
10^4 m	$1{,}5 \cdot 10^{-12}$

Obviously the imaginary part of the complex index of refraction needs to be rather small if we talk about "optics". If it would be **1,5**, i.e. about the same as the real part, the penetration depth would be 10^{-8} **m = 10 nm**—we have a rather opaque material in this case.

Calculate what that would mean in terms of *only* ε'' or *only* ε''.

From the definition of the complex index of refraction we have $2\kappa^2 = (\epsilon'^2 + \epsilon''^2)^{½} - \epsilon'$.

- For small ε'' we can develop the square root into a series and get $(\epsilon'^2 + \epsilon''^2)^{½} \approx \epsilon' + \epsilon''/2\epsilon'$.
- This leads to $\epsilon'' = 4\kappa^2 \cdot \epsilon'$. For a reasonable $\epsilon' = 2$ (giving **n = 1,4**) we have $\epsilon'' = 1{,}8 \cdot 10^{-15}$; $1{,}8 \cdot 10^{-29}$, $1{,}8 \cdot 10^{-23}$ for the three κ values from above.

To relate κ to the static conductivity σ_{static} we use the equation $\sigma_{DK}/\,2\epsilon_0\omega = n\kappa$, which gives

- $\sigma_{DK} = 2n\kappa\epsilon_0\omega = \kappa \cdot 2{,}8 \cdot 10^{16} \cdot 8{,}854 \cdot 10^{-12}\ s^{-1}A{\cdot}s{\cdot}V^{-1}m^{-1} = \kappa \cdot 2{,}48 \cdot 10^5\ (\Omega m)^{-1}$
- Plugging in the numbers for κ we get

κ	σ_{DK} [Ωm^{-1}]	$\rho_{DK} = 1/\sigma_{DK}$ [Ωcm]
$1{,}5 \cdot 10^{-8}$	$3.72 \cdot 10^{-3}$	$2{,}69 \cdot 10^4$
$1{,}5 \cdot 10^{-10}$	$3.72 \cdot 10^{-5}$	$2{,}69 \cdot 10^6$
$1{,}5 \cdot 10^{-12}$	$3.72 \cdot 10^{-7}$	$2{,}69 \cdot 10^8$

- For large penetration depth we need pretty good DC insulators. For **100 km =** 10^5 **m**, something a fibre optic cable should do, we have about **3 GΩcm**, a number that is not too large for good insulators like glass. Of course, the static resistivity is not the only reason for absorption.

- Discuss the results with respect to the complex index of refractions of **Si** and the dielectric function of **GaAs** as given in this link for frequencies above and below the band gap (after you located the band gap by straight thinking)

- First we look at the GaAS curves with a linear scale.
  - For energies below the bandgap, GaAs is transparent and both κ and ε'' should be zero. That's the case for energies **< 1.4 eV** as easily seen in the *linear* GaAs diagrams with the **eV** scale. The correct value is **1,42 eV**, so optical measurements do provide easy access to bandgaps.
- Or do they?
  - The log scale picture for Si is better suited to look for precise numbers. First, we need to convert wavelengths to **eV** via $\boldsymbol{E = h\nu = hc/\lambda}$. A big drop in κ for Si occurs around **400 nm** , corresponding to $\mathbf{(4{,}1356 \cdot 10^{-15}\ eVs \cdot 3 \cdot 10^{8}\ ms^{-1})/400 \cdot 10^{-9}\ m = 3{,}1\ eV}$ **????**
  - Aha! We are far off the proper value of **1,1 eV**. Obviously the proper wavelength must be about twice as large, **847 nm** to be exact. That is still in the infrared as is should be since **Si** wafers are not transparent to any visible wavelength.
  - What we learn is that the bandgap does show up very clearly in optical measurement (look at the linear scales!) but that precision measurements might be more tricky than naively expected.