Solution to Exercise 5.2-1: Fresnel Coefficients

- Consider an unpolarized light beam hitting the surface of an isotropic glass with n = 1,5 at right angles ($\alpha = 0^{\circ}$).
- Derive the simplified Fresnel equations as given in the script for $\alpha = 0^{\circ}$
- Setting the cosines to 1 in the TE or TM case gives

$$E_{ref} = E_{in} \cdot \frac{\sin \beta - \sin \alpha}{\sin \beta + \sin \alpha}$$

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$$E_{ref} = E_{in} \cdot \frac{n-1}{n+1}$$

$$q.e.d$$

- There is thus no difference between the TE and TM case.
- How much of the light will be reflected?
- We can use the simple equations for perpendicular incidence from above, re-written for intensities:

$$\frac{I_{\text{ref}}}{I_{\text{in}}} = \left(\frac{n-1}{n+1}\right)^2$$

The intensity of the reflected light is thus

$$I_{\text{ref}} = I_{\text{in}} \left(\frac{n-1}{n+1} \right)^{2}$$

$$= I_{\text{in}} \left(\frac{0.5}{2.5} \right)^{2}$$

$$= 0.04 I_{\text{in}}$$

- The reflected intensity is thus 4 % of the incoming intensity.
- What is the phase relation between incoming, reflected and transmitted light?
- To answer that question we must look at the field strength. There is a *minus sign* and the the phase of the reflected beam thus is phase-shifted by **180** $^{\circ}$ = π
- How does the beam leave the crystal (Intensity and polarization)?

- For the transmitted beam we have the simple relation $I_{tr} = I_{in} I_{re}$. The intensity of transmitted beam thus is 96 % of the incoming intensity for an optical material with n = 1,5.
 - At the "exit" from the *n* = 1,5 optically dense material to the *n* = 1 less dense material α and β interchange their role or sinβ/sinα = *n* = 1,5 now. Taking this into account we get the same equations for the perpendicular incidence as above but without the "-" (minus) sign for the field strengths. This means that 96 % of 96 % (= 92,16 %) of the incoming beam exits the optical material.
 - How about phases? Looking at the original <u>Fresnel equations</u> for the electrical field strength we see that in both the **TE** and **TM** case there is no sign and therefore phase jump for the *transmitted* wave for all angles (since α , β <= 90 ° in all cases). For the reflected wave at the optically less dense medium (the wave reflected back into the interior of the optical material) there isn't a phase change either because the minus sign is no longer there.
- Now consider these questions for some polarization of the incoming light.
- No difference since we have the same equations for both the **TE** and **TM** case.