Solution to Exercise 5.1-2 Energy, Field strength and Photons

Illustration

You have a LED as a light source that emits a monochromatic light beam with wave length (in air) of **λ = 500 nm**. The light is generated in a very small volume ("point source") and spreads out in a cone that illuminates a circle with radius **1 cm** at a distance of **10 cm** on some white paper. The **LED** has an over-all or plug efficiency of **50 %** and is driven at **2V** with **20 A**.

Question 1: How much power in **W/m^2** flows into the paper?

Total power = **UI = 40 W**
Light power = **50 %** of total power = **20 W**
Light flux = **20 W / πr^2 = 6,37 W/cm^2**
With **eV** instead of **Ws = J** and **1 J = 1 Ws = 6,24 · 10^{18} eV** we have
Light flux = **3,97 · 10^{19} eV / s · cm^2**.

Question 2: How does that number compare with the light power coming form the sun at "AM 1" conditions (High noon, equation, no clouds)? You're supposed to know this basic number in some "simple number approximation".

The sun at a cloudless day at high noon at the equator delivers about **1 kW/m^2 = 1000 / 10000 W/cm^2 = 0,1 W/cm^2**
Our LED thus delivers a very high (and unrealistic) intensity of **63,7** times more than the sun.

Question 3: How many photons per second must hit the piece of paper if we discuss the energy flux now in the particle picture?

A wavelength of **500 nm** corresponds to a photon with energy **$h\nu = hc/\lambda$ = {(4,1356 · 10^{-15}) · (3 · 10^{17})} / 500 eVs · nms^{-1} · nm^{-1} = 2,48 eV**. The necessary **3,97 · 10^{19} eV/scm^2** thus corresponds to **3,97 · 10^{19} / 2,48 = 1,60 · 10^{19} photons/scm^2** Since we need to illuminate an area of **3,14 cm^2,** we need **5,03 · 10^{19} photons/s**

Question 4: What kind of field strength would we have on the paper? Consider first that the light beam is fully coherent, next that the photons are completely uncorrelated.

The energy flux in the light beam is given by
$\langle S \rangle = \frac{1}{2} E_0 H_0 = (E_0)^2/(Z_W)$ = 6,37 W/cm^2

- For the electrical field strength in a fully coherent wave we have
  $E_0 = [Z_W \cdot 6{,}37\ \mathrm{VW/Acm^2}]^{1/2} = [377 \cdot 6{,}37\ \mathrm{V^2/cm^2}]^{1/2} = 49\ \mathrm{V/cm}$
  That is a rather low field strength.
- A completely incoherent light consists of waves with all kinds of phases and all kinds of directions of the electrical field vectors. The total field strength then is a vector sum that tends to average to zero.
  We might assume that the number of independent "waves" equal the number of photons. That gives the field strength per wave = photon to **E_{Ph} = 49 / 5,03 · 10^{19} V/cm = 9,74 · 10^{-19} V/cm**. That is, of course, a rather meaningless number.
- If we consider that a photon delivers an energy of **2 eV** to an area of about **1 µm^2** within **1 ns**, we would get **$E_{Ph} \approx$ 3,5 V/cm**, which is more like it but still more or less nonsense.
- How about assuming that the *energy* **W_{Ph}** of a photon (= **2,5 eV**, for example) is contained in a volume of **λ^3 (1 µm)^3**, for example). We then have roughly **$W_{Ph} = (\frac{1}{2}\epsilon_0 \cdot E^2)/\lambda^3$** from the relation between energy density and field strength ***E***.
  Going through the numbers we obtain **$E_{Ph} \approx$ 3.000 V/cm**. That is a number one could live with.

Question 5: What does the number of photons produced per second tell you about recombination rates, carrier densities, and current densities in the semiconductor?

We need at least a recombination rate of **R = 5,03 · 10^{19} s^{-1}**between electrons and holes that produces light. If we loose some of the light, the rate must be higher. The recombination ot the carriers take place in a device volume **V_{Dev}** given by lateral area **F** of the device times length **l_{Rec}**of the recombination zone; **$V_{Dev} = F \cdot l_{Rec}$**.

- We can always express the specific recombination rate per **cm^{-3}** by **R = 5,03 · 10^{19}/ s · $F \cdot l_{Rec} = n/\tau$** with ***n*** = surplus carrier density, **τ** = carrier life time $\approx$ **1 ns**.
- The necessary carrier density that must be supplied by the current is thus **$n = R\tau / F \cdot l_{Rec}$**. If we assume **$F$ = 10^{-4} cm^2**, **τ = 10^{-9} s**; we have **n = 5,03 · 10^{18} m^{-3}** which looks reasonable.

The total current ***I*** was **20 A**. The current *density* ***j*** for a device with cross-sectional area ***F*** of **10^{-4} cm^{2}cm^2** is
j = 2 · 10^5 A/cm^2 which is a bit on the high side..

- The number N_e of electrons (or holes "on the other side") that we inject is $\mathbf{N_e = 20\ (C/s) / 1{,}6 \cdot 10^{-19}\ C = 1{,}25 \cdot 10^{20}\ s^{-1}}$. If half of this electrons recombine and produce light (we assumed an efficiency of **50 %**), we have $\mathbf{6{,}25 \cdot 10^{19}}$ electrons per second available for this, a number that matches quite *fortuitously* with the required $\mathbf{5{,}03 \cdot 10^{19}\ s^{-1}}$. Or maybe, it's not that *accidental*?