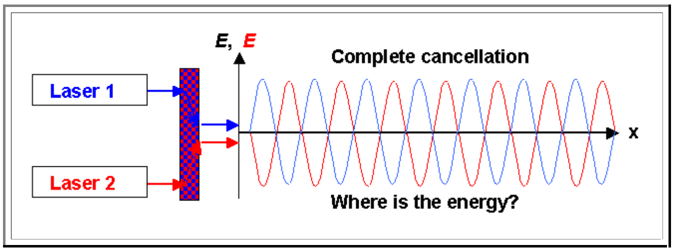
Interference Paradox

Advanced

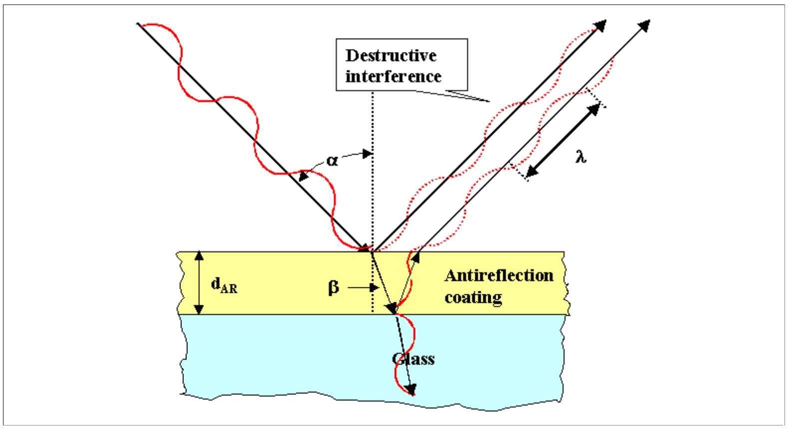
- Consider two Laser beams ideally described by $\underline{E} = \underline{E}_0 \exp(\underline{k}x - \omega t)$ but with phase differences of **180°** or π.
  - With some device (semitransparent mirror, prisms, ...) we get both light beams to travel along the same ***x***-direction as shown below. They must now cancel each other completely. There is no light anymore along the x-direction!



  - We have a big paradox:

Where is the energy contained in the two Laser beams that are still emitted by the running Lasers?

- If you find the situation a bit artificial and not related to practical live consider this question:
  - There are earphones already on the market where an acoustic signal generated "live" and exactly in antiphase to some signal measured - e.g. the noise in an airplane - cancels that signal and thus you do not hear the noise anymore. Similar but not the same because the acoustic energy contained in the cabin noise is still mostly there.
  - So how about large-scale development efforts to cancel the noise in modern cars by generating "anti noise" and beaming it into the inside by loudspeakers. The noise generated by the engine and whatever else contains energy, and so does the anti-noise generated by the speaker system. But if the system works it is now quiet and the acoustic energy has "disappeared". Since energy cannot disappear, the question is: where is it?
- Let's inch towards the answer. Despite some claims to the opposite, there is no easy answer. Consult this link, for example.
  - Let's look at a *real* situation where the paradox apprears to some extent: **antireflection coatings** as shown below.



Schematic view (don't look too closely at phases) of the working of an antireflection coating

  - By putting a dielectric layer with the right index of refraction ***n*** - thickness $\boldsymbol{d_{AR}}$ combination on top of, e.g., a piece of glass, the two reflected beams - one at the **AR** coating - air interface, one from the **AR** coating - glass interface - exactly cancel each other. There is thus no reflected beam; it isn't called antireflection coating for nothing.
  - Where is now the energy no longer contained in the non-existent reflected beam? In the transmitted beam, of course - where else? That's why you see better through lenses with an antireflection coating: more light reaches your eye.
- Clear - but note that we have not really *proved* that statement!
  - The interference paradox essentially *challenges* the validity of the energy conservation law. The argumentation above, however, is based on energy conservation being always valid.

    - Nevertheless, the statement that the energy cancelled by destructive interference is now somewhere else is generally true because it can be *proved* that energy conservation still obtains!
- In the top picture the crucial part is the "*some device (semitransparent mirror, prisms, ...)*" brings the beams together.
    - There is *no* device that can do that without reflecting some of the intensity. If you get the case of fully destructive interference to the right, you will have to produce a lot of reflection to the left. In other constructions you might even influence what the light sources do, up to inducing self-destruction of the Lasers (e.g. by pointing two Lasers against each other).
- But again; it is one thing to make these statements and another thing to prove them by calculating the various intensities. If you want to dig deeper, consider the following case.
    - Two simple dipole antennas emitting radio waves are put close together (distance much smaller than the wavelength). You feed both antennas with the same signal but with an adjustable phase difference. This is technically easy to do.
    - For a phase difference of π the emitted waves would cancel. The situation is exactly the same as shown above except that it could be done easily. The problem is that we have no mirrors or gadgets now that can take the "blame" as in the example above. This case, however, can be calculated in detail.
    - What one finds out by going through the proper equations is that antennas do not just emit, they also "receive". There is always some coupling expressed in some *impedance* that is not constant but depends on what the two antennas do. Important terms like *near-field* and *far-field* come into their own, and in the end - well, read the article yourself!