4.3.2 Beyond Mean Field Theory

Some General Considerations
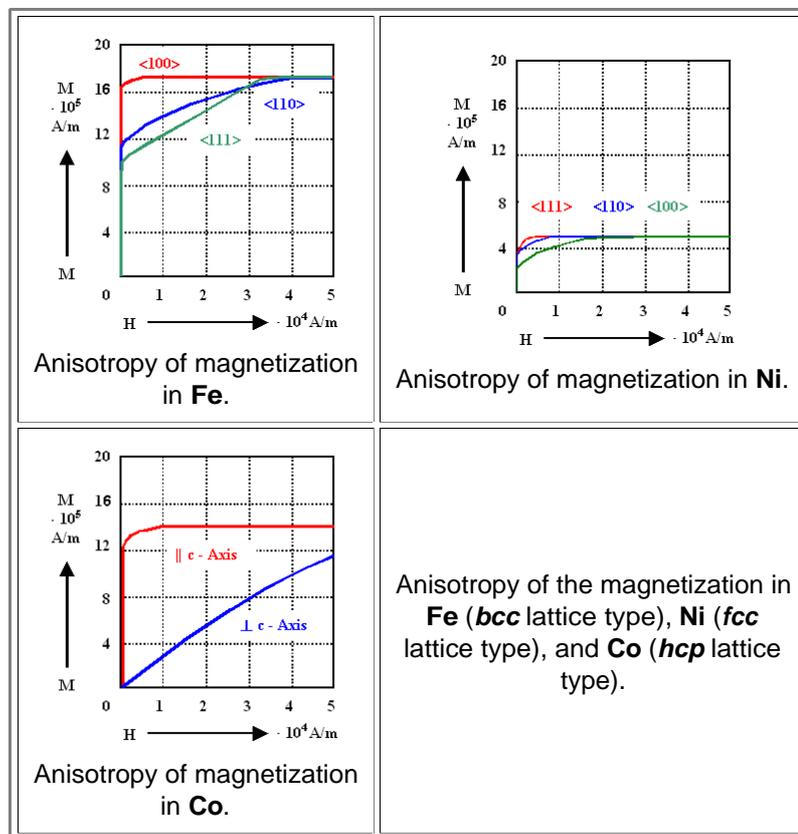
According to the mean field theory, if a material is ferromagnetic, *all* magnetic moments of the atoms would be coupled and point in the same direction. We now ask a few questions:

- **1.** *Which direction* is that going to be for a material just sitting there? Is there some preferred internal direction or are all directions equal? In other words: Do we have to make the fictitious Weiss field H_{Weiss} larger in some directions compared to other ones? Of course, we wonder if some crystallographic directions have "special status".
- **2.** What happens if an external field is superimposed in some direction that does *not* coincide with a preferred internal direction?
- **3.** What happens if it *does*? Or if the external field is *parallel* to the internal one, but pointing in the *opposite* direction?

The (simple) mean field theory remains rather silent on those questions. With respect to the first one, the internal alignment direction would be determined by the direction of the fictive field H_{Weiss}, but since this field does not really exist, each direction seems equally likely.

- In real materials, however, we might expect that the direction of the magnetization is not totally random, but has some specific preferences. This is certainly what we must expect for *crystals*.
- A specific direction in real ferromagnetic materials could be the result of crystal anisotropies, inhomogeneities, or external influences - none of which are contained within the mean field theory (which essentially treats perfectly isotropic and infinitely large materials).

Real ferromagnetic materials thus are more complicated than suggested by the mean field theory - for a very general reason:

Even if we can lower the internal *energy* U of a crystal by aligning magnetic moments, we still must keep in mind that the aim is always to minimize the *free enthalpy* $G = U - TS$ of the *total* system.

- While the *entropy* part coming from the degree of orderliness in the system of magnetic moments has been taken care of by the general treatment in the frame work of the orientation polarization, we must consider the enthalpy (or energy) U of the *system* in more detail. So far we only minimized U with respect to *single* magnetic moments in the Weiss field.
- This is so because the mean field approach essentially relied on the fact that by aligning the spins relative to the (fictitious) Weiss field, we lower the energy of the individual spin or magnetic moments as treated before by some energy W_{align}. We have

$$U_{align} = U_{random} - W_{align}$$

- But, as discussed above, real materials are mostly (poly)crystals and we must expect that the real (quantum-mechanical) interaction between the magnetic moments of the atoms are different for different directions in the crystal. There is some anisotropy that must be considered in the U_{align} part of the free enthalpy.
- Moreover, there are other contributions to U not contained in the mean field approach. Taken everything together makes quantitative answers to the questions above exceedingly difficult.

There are, however, a few relatively simple general rules and experimental facts that help to understand what really happens if a ferromagnetic material is put into a magnetic field. Let's start by looking at the **crystal anisotropy**.

Crystal Anisotropy

Generally, we must expect that there is a preferred crystallographic direction for the spontaneous magnetization, the so-called "**easy directions**". If so, it would need some energy to change the magnetization direction into some other orientations; the "**hard directions**".

- That effect, if existent, is easy to measure: Put a single crystal of the ferromagnetic material in a magnetic field H that is oriented in a certain crystal direction, and measure the magnetization of the material in that direction:
- If it happens to be an *easy* direction, you should see a strong magnetization that reaches a **saturation** value - obtained when all magnetic moments point in the desired direction - already at low field strength H. If, on the other hand, H happens to be in a *hard* direction, we would expect that the magnetization only turns into the H direction *reluctantly*, i.e. only for large values of H will we find saturation.

This is indeed what is observed, classical data for the elemental ferromagnets **Fe, Ni, Co** are shown below:

Anisotropy of magnetization in **Fe**.

Anisotropy of magnetization in **Ni**.

Anisotropy of magnetization in **Co**.

Anisotropy of the magnetization in **Fe** (*bcc* lattice type), **Ni** (*fcc* lattice type), and **Co** (*hcp* lattice type).

The curves are easy to interpret qualitatively along the lines stated above; consider, e.g., the **Fe** case:

- For field directions not in **<100>**, the spins become aligned in the **<100>** directions pointing as closely as possible in the external field direction.
- The magnetization thus is just the component of the **<100>** part in the field direction; it is obtained for arbitrarily small external fields.
- Increasing the magnetization, however, means turning spins into a "hard" directions, and this will proceed reluctantly for large magnetic fields.
- At sufficiently large fields, however, all spins are now aligned into the external field directions and we have the same magnetization as in the easy direction.

The curves above contain the material for a simple little exercise:

Exercise 4.3-2

Magnetic moments of Fe, Ni, Co

Questionaire

Multiple Choice questions to 4.3.2