Illustration

Solution to Exercise 3.2-1

- Sometimes, a question can be more tricky than originally intended. That is the case here - lets see why.
  - First lets get the **e.s.u** out of the way. It means "*electrostatic units*" which are sub-units of the old c.g.s (centimeter-gram-second) system, and still much in use.
  - Few things are more confusing than converting electric or magnetic **c.g.s.** units into the SI (Standard International) kilogram- meter- second-Ampère system. If you are not somewhat familiar with that, read up the basic modules accessible by the links to this topic.
- In the case given here, you have to multiply with $|c|/10 = 3{,}3356 \cdot 10^{-10}$ (**c =** vacuum speed of light) to obtain the charge in **[C]** (The magnitude signs | | simply mean hat you only take the number!); and since the dipole moment is charge times distance, the distance in *e.s.u* units must be **cm**.
  - We obtain
    $\mu_{water} = 1{,}87 \cdot 10^{-18}.\ 3{,}3356 \cdot 10^{-10}\ C \cdot cm = 6{,}24 \cdot 10^{-28}\ C \cdot cm$
  - Lets see if that is reasonable: A water molecule carries about one elementary charge $= 1{,}6 \cdot 10^{-19}$ **C** at the end of the dipole, and the distance will be about **1 Å =** 10^{-8} **cm**. This would give a dipole moment of $1{,}6 \cdot 10^{-27}$ **C·cm**, so the number we got should be correct

- Now to the tricky part. First it is important to realize that:
  - A material with *completely oriented natural dipoles* does not have a dielectric constant ϵ_r or dielectric susceptibility $\chi = \epsilon_r - 1$ anymore!
  - Consider: χ was the proportionality factor between the external field ***E*** and the induced polarization ***P***

$$P = \epsilon_0 \cdot \chi \cdot E$$

  - If the field doubles, the polarization, and thus the degree of orientation into the field doubles.
  - However, if *all* dipoles are *fully aligned*, the polarization is at a maximum and will not respond to the field anymore; χ looses its meaning.
- Nevertheless, we could take this fully polarized material, stick it into a plate capacitor, and just measure how the capacitance **C** changes . This would give us a value for ϵ_r simply by computing C_{after}/C_{before}. Lets see if we can do this.
  - For the capacity *before* we use our fully polarized dielectric we have with some applied voltage ***U*** and some corresponding charge Q_0

$$C_{before} = \frac{Q_0}{U}$$

  - For the capacity *after* we use our fully polarized dielectric we have
    $C_{after} = (Q_0 + Q_{pol})/U$,
    and this gives us

$$\frac{C_{after}}{C_{before}} = \epsilon_r = \frac{Q_0 + Q_{pol}}{Q_0}$$

- This does not help, however, because we do not know Q_0. Lets try a different approach and look at $C_{after} - C_{before}$.
  - We obtain .

$$C_{after} - C_{before} = \epsilon_r \cdot C_{before} - C_{before} = C_{before} \cdot (\epsilon_r - 1) = C_{before} \cdot \chi = \frac{Q_0 + Q_{pol}}{U} - \frac{Q_0}{U} = \frac{Q_{pol}}{U}$$

$$\chi = \frac{Q_{pol}}{U \cdot C_{before}} = \frac{Q_{pol}}{Q_0}$$

- This looks better, but it is still not useful - we do not know Q_0. We still have the same problem: The changes are not *proportional* to what we had *before* the introduction of the dielectric, but *absolute* - we are, in effect, adding a fixed charge and thus switching a second capacitor in series.

Lets try a different approach. We know that $\chi(H_2O) \approx 80$. The polarization that goes with this value increases steadily as the field strength inducing the polarization increases - as long as we have $P = \chi \cdot E$

- For large field strength, however, this "law" must break down - we reach the absolute limit of polarization sooner or later.
- So lets compute in a first approximation the field strength needed (within the simple law) to induce the maximum polarization and compare the value obtained to field strengths usually encountered.

First, we compute the maximum polarization P_{max}. This is simply the the charge q_{H2O} on one end of the water dipole times the distance of the charges d_{H2O} divided by the (area) density of the dipoles, i.e. the (area density) of water.

- The dipole moment of water is given by

$$\mu_{water} = q_{H2O} \cdot d_{H2O} = 1{,}87 \cdot 10^{-18} \cdot 3{,}3356 \cdot 10^{-10}\ \mathrm{C \cdot cm}$$

- We need d_{H2O} to compute q_{H2O}; from the picture in the question we find it to be $d_{H2O} = 0{,}0958\ \mathrm{nm} \cdot \cos(104{,}45^o/2) = 0{,}0586\ \mathrm{nm}$.
- The (effective) charge q_{H2O} at the end of a dipole thus is

$$q_{H2O} = q_{H2O} = 6{,}24 \cdot 10^{-28}\ \mathrm{C \cdot cm} / 0{,}0586 \cdot 10^{-7}\ \mathrm{cm} = 1{,}065 \cdot 10^{-19}\ \mathrm{C}$$

about 2/3 of an elementary charge.

The density of water is $\rho_{H2O} = 1\ \mathrm{kg/l} = 1\mathrm{g/cm}^3$ by definition.

- One mol of water is **1+ 1+ 16 = 18 g** which tells us that we have **1 mol = 6.022 · 10^{23}** water molecules in **18 cm^3**.
- The areal density ρ_{areal} of dipoles is therefore

$$\rho_{areal} = \frac{6.022 \cdot 10^{23} \cdot 0.0586\ \mathrm{nm}}{18\ \mathrm{cm}^3} = 1{,}96 \cdot 10^{14}\ \mathrm{dipoles/cm}^2$$

- Converting *volume* densities to **areal** or **surface densities** may appear tricky. If you are not sure about how it is done, consult the link.
- The maximum polarization P_{max} thus is.

$$P_{max} = 1{,}065\ 10^{-19} \cdot 1{,}96 \cdot 10^{14}\ \mathrm{C/cm}^2 = 2{,}087 \cdot 10^{-5}\ \mathrm{C/cm}^2$$

- If we want to generate this polarization with an electrical field and a susceptibility $\chi = 80$, we need a saturation field strength E_{sat} of

$$E_{sat} = P_{max}/80 \cdot \epsilon_0 = 2{,}087 \cdot 10^{-5}/80 \cdot 8{,}854 \cdot 10^{-12}(\mathrm{C/cm}^2) \cdot (\mathrm{Vm/C}) = 2{,}946 \cdot 10^6\ \mathrm{V/cm}$$

OK, that is a definite result. Now we have to ask ourselves, how we must compare a field strength of about **3 ·10^6 V/cm** to "normal" field strengths.

- To some extent, we do that in sub-chapter 3.5.1, but common sense tells us that we would certainly use **1mm** or more of a dielectric to insulate a wire carrying **1000 V**, for example. This translates to a "typical" field strength of 1**0.000V/cm**.
- Many materials will be destroyed at field strengths of very roughly **100.000 V/cm**, so **3 · 10^{6}V/cm** is very large, indeed.
- However, dielectrics in integrated circuits must be able to operate at field strength of this order of magnitude. Take **3 V** and a thickness of the dielectric of **10 nm** - a not atypical combination - and you have a field strength of **3 · 10^{6}V/cm**, just what we calculated.

Anyway, if we take **100.000 V/cm** as a "normal value, we realize that the only **3,4%** of the dipoles need to be oriented in field direction, whereas the rest could be oriented at random. (**1 · 10^5/2,946 ·10^6 = 0,034**).

- This is not the physical reality, of course. A more physical interpretation is that all dipoles change whatever orientation they happen to have by about **3,4 %** in field direction. What that means precisely, we will leave open, the general meaning, however, is clear:
- The effect of polarization would hardly be noticeable by just looking at the distribution of the dipoles. It is a rather small effect, even for a material with a comparatively very large dielectric susceptibility.