Solution to Exercise 2.1-2

Derive numbers for v_0, v_D, τ, and l

First Task: Derive a number for v_0 (at room temperature). We have

$$v_0 = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{8{,}6 \cdot 10^{-5} \cdot 300}{9{,}1 \cdot 10^{-31}} \; \frac{\text{eV} \cdot \text{K}}{\text{K} \cdot \text{kg}}\right)^{1/2} = 1{,}68 \cdot 10^{14} \cdot \left(\frac{\text{eV}}{\text{kg}}\right)^{1/2}$$

- The dimension "square root of **eV/kg**" does not look so good - for a velocity we would like to have **m/s**. In looking at the energies we equated kinetic energy with the classical dimension $[\text{kg} \cdot \text{m}^2/\text{s}^2] = [\text{J}]$ with thermal energy kT expressed in **[eV]**. So let's convert **eV** to **J** (use the link) and see if that solves the problem. We have $1 \text{ eV} = 1{,}6 \cdot 10^{-19} \text{ J} = 1{,}6 \cdot 10^{-19} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ which gives us

$$v_0 = 1{,}68 \cdot 10^{14} \cdot \left(\frac{1{,}6 \cdot 10^{-19} \text{ kg} \cdot \text{m}^2}{\text{kg} \cdot \text{s}^2}\right)^{1/2} = 5{,}31 \cdot 10^4 \text{ m/s} = 1{,}91 \cdot 10^5 \text{ km/hr}$$

Possibly a bit surprising - those electrons are no sluggards but move around rather fast. Anyway, we have shown that a value of $\approx 10^4$ **m/s** as postulated in the backbone is really OK.

- Of course, for $T \to 0$, we would have $v_0 \to 0$ - which should worry us a bit ???? If instead of room temperature ($T = 300$ K) we would go to let's say **1200 K** , we would just double the average speed of the electrons.

Second Task: Derive a number for τ. We have

$$\tau = \frac{\sigma \cdot m}{n \cdot e^2}$$

First we need some number for the concentration of free electrons per m^3. For that we complete the table given, noting that for the number of atoms per m^3 we have to divide the density by the atomic weight.

Atom	Density $[\text{kg} \cdot \text{m}^{-3}]$	Atomic weight $\times 1{,}66 \cdot 10^{-27}$ kg	Conductivity σ $\times 10^5 [\Omega^{-1} \cdot \text{m}^{-1}]$	No. Atoms $[\text{m}^{-3}]$ $\times 10^{28}$
Na	970	23	2,4	2,54
Cu	8.920	64	5,9	8,40
Au	19.300	197	4,5	5,90

- So let's take $5 \cdot 10^{28}\ \text{m}^{-3}$ as a good order of magnitude guess for the number of atoms in a m^3, and for a first estimate some average value $\sigma = 5 \cdot 10^5\ [\Omega^{-1} \cdot \text{m}^{-1}]$. We obtain

$$\tau = \frac{5 \cdot 10^5 \cdot 9{,}1 \cdot 10^{-31}}{5 \cdot 10^{28} \cdot (1{,}6 \cdot 10^{-19})^2} \; \frac{\text{kg} \cdot \text{m}^3}{\Omega \cdot \text{m} \cdot \text{A}^2 \cdot \text{s}^2} = 3{,}55 \cdot 10^{-16} \; \frac{\text{kg} \cdot \text{m}^2}{\text{V} \cdot \text{A} \cdot \text{s}^2}$$

Well, somehow the whole thing would look much better with the unit **[s]**. So let's see if we can remedy the situation.

- Easy: Volts times Amperes equals *Watts* which is power, e.g. energy per time, with the unit $[\text{J} \cdot \text{s}^{-1}] = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$. Insertion yields

$$\tau = 1{,}42 \cdot 10^{-28} \frac{kg \cdot m^2 \cdot s^3}{kg \cdot m^2 \cdot s^2} = 3{,}55 \cdot 10^{-16}\ s = 0.35\ fs$$

- The backbone thus is right again. The scattering time is in the order of femtosecond which is a short time indeed. Since all variables enter the equation linearly, looking at somewhat other carra ier densities (e.g. more than **1** electron per atom) or conductivities does not really change the general picture very much.

- Third Task: Derive a number for $\mathbf{v_D}$. We have (for a field strength $\boldsymbol{E}$ **= 100 V/m = 1 V/cm**)

$$|v_D| = \frac{E \cdot e \cdot \tau}{m} = \frac{100 \cdot 1{,}6 \cdot 10^{-19} \cdot 3{,}55 \cdot 10^{-16}}{9{,}1 \cdot 10^{-31}} \frac{V \cdot C \cdot s}{m \cdot kg} = 6{,}24 \cdot 10^{-3} \frac{V \cdot A \cdot s^2}{m \cdot kg}$$

$$= 6{,}24 \cdot 10^{-3} \frac{kg \cdot m^2 \cdot s^2}{m \cdot kg \cdot s^3} = 6{,}24 \cdot 10^{-3}\ m/s = 6{,}24\ mm/s$$

- This is somewhat larger than the value given in the backbone text.
  - However - a field strength of **1 V/cm** applied to a *metal* is huge! Think about the current density $\boldsymbol{j}$ you would get if you apply **1 V** to a piece of metal **1 cm** thick.
  - It is actually $\boldsymbol{j} = \sigma \cdot \boldsymbol{E} = 5 \cdot 10^7\ [\Omega^{-1} \cdot m^{-1}] \cdot 100\ V/m = 5 \cdot 10^9\ A/m^2 = 5 \cdot 10^5\ A/cm^2$!
  - For a more "reasonable" current density of $\mathbf{10^3\ A/cm^2}$ we have to reduce $\boldsymbol{E}$ hundredfold and then end up with $\mathbf{|v_D| = 0{,}0624\ mm/s}$ - and that is slow indeed!

- Fourth Task: Derive a number for $\boldsymbol{l}$. We have

$$l_{min} = 2 \cdot v_0 \cdot \tau = 2 \cdot 5{,}31 \cdot 10^4 \cdot 3{,}55 \cdot 10^{-16}\ m = 3{,}77 \cdot 10^7\ m = 0{,}0377\ nm$$

  - Right again! If we add the comparatively miniscule $\mathbf{v_D}$, nothing would change. Decreasing the temperature would lower $\boldsymbol{l}$ to eventually zero, or more precisely, to $\mathbf{2 \cdot v_D \cdot \tau}$ and thus to a value far smaller than an atom..