Solution to Exercise 2.1-1

Derive and Discuss numbers for μ

First Task: Derive numbers for the mobility **μ**.

- First we need typical conductivities and electron densities in *metals*, which we can take from the table in the link.
- At the same time we expand the table a bit

Material	ρ [Ω cm]	σ [Ω⁻¹ cm⁻¹]	Density $d \times 10^3$ [kg m⁻³]	Atomic weight w [× 1u = 1,66 · 10⁻²⁷ kg]	$n = d/w$ [m⁻³]
Silver Ag	$1{,}6 \cdot 10^{-6}$	$6.2 \cdot 10^5$	10,49	107,9	$5{,}85 \cdot 10^{28}$
Copper Cu	$1{,}7 \cdot 10^{-6}$	$5.9 \cdot 10^5$	8,92	63,5	$8{,}46 \cdot 10^{28}$
Lead Pb	$21 \cdot 10^{-6}$	$4.8 \cdot 10^4$	11,34	207,2	$3{,}3 \cdot 10^{28}$

For the mobility **μ** we have the equation

$$\mu = \frac{\sigma}{q \cdot n}$$

With **q** = elementary charge **= 1,60 10⁻¹⁹ C** we obtain, for example for **μ$_{Ag}$**

$$\mu_{Ag} = \frac{6{,}2 \cdot 10^5}{1{,}6 \cdot 10^{-19} \cdot 5{,}85 \cdot 10^{28}} \frac{m^3}{C \cdot \Omega \cdot cm} = 66{,}2 \frac{cm^2}{C \cdot \Omega}$$

The unit is a bit strange, but remembering that **[C] = [A · s]** and **[Ω] = [V/A]**, we obtain

$$\mu_{Ag} = 66{,}2 \frac{cm^2}{Vs}$$

$$\mu_{Cu} = 43.6 \frac{cm^2}{Vs}$$

$$\mu_{Pb} = 9{,}1 \frac{cm^2}{Vs}$$

Second Task: Derive numbers for the drift velocity v_D by considering a reasonable field strength.

● The mobility **µ** was defined as

$$\mu \;=\; \frac{v_D}{E}$$

or

$$v_D \;=\; \mu \cdot E$$

So what is a reasonable field strength in a metal?

● Easy. Consider a cube with side lengt **l = 1 cm**. Its resistance **R** is given by

$$R \;=\; \frac{\rho \cdot l}{F} \;=\; \rho \;\Omega$$

● A **Cu** or **Ag** cube thus would have a resistance of about **1,5 ·10^{-6} Ω.** Applying a voltage of **1 V**, or equivalently a field strength of **1 V/cm** thus produces a current of **$l = U/R \approx$ 650 000 A** or a current density **j = 650 000 A/cm^2**

● That seems to be an awfully large current. Yes, but it is the kind of current *density* encountered in integrated circuits! Think about it!

● Nevertheless, the wires in your house carry at most about **30 A** (above that the fuse blows) with a cross section of about **1 mm^2**; so a reasonable current density is **3000 A/cm^2**, which we will get for about **U = 1,5 ·10^{-6} Ω · 3000 A = 4,5 mV**.

● For a rough estimate we then take a field strength of **5 mV/cm** and a mobility of **50 cm^2/Vs** and obtain

$$v_D \;=\; 50 \cdot 5 \;\frac{mV \cdot cm^2}{cm \cdot V \cdot s} \;=\; 0{,}25 \;\frac{cm}{s} \;=\; 2{,}5 \;\frac{mm}{s}$$

That should come as some surprise! The electrons only have to move v e r y s l o w l y *on average* in the current direction (or rather, due to sign conventions, against it).

● Is that true, or did we make a mistake?

● It *is* true! However, it does *not* mean, that electrons will not run around like crazy inside the crystal, at very high speeds. It only means that their *net* movement in current anti-direction is very slow.

● Think of an single fly in a fly swarm. Even better read the module that discusses this analogy in detail. The flies are flying around at high speed like crazy - but the fly swarm is not going anywhere as long as it stays in place. There is then no drift velocity and no net fly current!