False paradoxes of superposition in electric and acoustic waves

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When acoustic or electromagnetic waves cancel by destructive interference, the wave impedance reflected to the sources of the wave energy changes so that the input power is reduced correspondingly. Because this rather subtle point is not discussed in most instructional treatments of wave physics, misconceptions concerning the apparently "missing" energy are widespread, as reflected in semipopular science writing. This paper presents an analysis of the cases of destructive and constructive wave interference, and analogous problems for lumped electrical and mechanical systems.

INTRODUCTION

Some popular science and science fiction articles and stories have put on record a misconception formed by many students. When two confluent waves cancel by destructive interference, there is obviously no net energy transported through the region of cancelation. A question then arises concerning the disposition of the energy which was carried by each of the two waves in the absence of the other canceling wave. This question, however, contains an implicit misstatement.

By asking, "When light waves cancel, where does the energy go?," the question implicitly assumes that the same energy is carried by the two interfering waves when *both* are present as when they are each present separately. As such, it is a misuse of the principle of linear superposition. The total electric or magnetic field of a combination of waves can be found by addition of the fields due to each wave acting separately. However, the power carried by the wave, which is a product of field intensities, and the wave impedance, which is a ratio of intensities, can *not* be determined by superposition.

The well-known popular science writer, Asimov, once published an "explanation" of this question with the incorrect and rather vague assertion that the energy originally carried by the waves is "converted into heat." This catch-all phrase does not apply to many specific structures which one can analyze, as Asimov himself agreed in a private communication.[1] This point is not raised to carp at the generally excellent science writing of Asimov, but to show that the misconceptions described are widespread, and are, in fact, frequently expressed by science students.

The well-known science fiction author, Clarke (who probably also knows better), has written a short story in his book *Tales From the White Hart*, concerning a disgruntled scientist who secreted a "sound canceling" device in a concert hall.[2] This hypothetical device used a microphone to pick up sound from the stage, and then a powerful amplifier to produce a negative amplitude but corresponding sound from a concealed loudspeaker. Clarke makes the incorrect implication that one concealed loudspeaker can cancel sound throughout the entire auditorium, even though the original sources of sound are scattered across the stage and orchestra pit. However, given this initial exaggeration, he describes the surprise of the singers and musicians as they attempt to sing and play, unable to produce a sound from their throats or instruments. Once given the initial exaggeration, this scene is possible, but Clarke also describes a completely wrong inference at the conclusion of the story. He states that the concealed amplifier finally fails, and—based on the incorrect assumption that the singers and musicians, and cancelation amplifier, had all been producing acoustic power during their unsuccessful attempts to produce sound, and that this acoustic energy was somehow held in some kind of "suspended animation" until the cancelation amplifier went off—an explosive crash of sound fills the air and destroys the concert hall!

If nothing else, this author can assure Clarke's readers that nothing of the kind would happen, because both the singers, and the musicians, and the sound canceling amplifier produce *no* power while they all operate simultaneously. Air of oscillating velocity may emerge from the singer's mouth, or the F holes of the violin, or the end of a wind instrument. However, it is met by an incoming wave with equal and opposite sound pressure oscillations, which cancels the pressure oscillations associated with the emerging wave. We can describe the situation as one in which the space surrounding the singers and musicians *appears* to have zero acoustic impedance, although it is not in any way to be inferred that the presence of a "sound canceling" source modifies the acoustic properties of the air in any way. This apparent change in impedance (acoustically a pressure to velocity ratio, or electrically a voltage to current ratio) is explained more fully below.

ELECTRICAL EXAMPLE

Consider the physical situation described by Fig. 1. Two center-fed electrical dipole antennas, each of a length equal to $\frac{1}{2}$ wavelength of the sinusoidal current used, are physically parallel and very close together. Each is driven by a separate electrical current generator, but the two generators are locked into precise phase synchronization at either an in-phase or out-of-phase (i.e., 180° phase angle) condition. In practice, both antennas could either be directly connected at their centers to a common generator for in phase, or cross connected at the centers for an out-of-phase drive condition.

By the cavalier application of the knowledge that the equivalent electrical impedance of such a halfwave antenna is approximately 73 Ω, one may calculate that the power radiated into space by one antenna acting alone (the other current source set to zero) is:

$$P = I^2 73, \tag{1}$$

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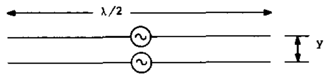
Fig. 1. Two center-fed half wave antennas, driven by in- or out-of-phase current sources. Their spacing y is very much smaller than a quarter wavelength.



where I is the (root-mean-square) value of the sinusoidal driving current. The same power is radiated by the second antenna acting alone. The false conclusion one might draw from this is that the power radiated by both antennas together is twice the power radiated by one alone, but even a relatively unsophisticated student knows that the power is proportional to I^2, and can reason thus: because the antennas are so close, the effect of driving both in phase is equivalent to a single antenna with a total current equal to the sum of the two individual currents. Therefore, operating both antennas in phase doubles both the radiated magnetic *and* electric fields, and consequently quadruples the power.

That reasoning is, indeed, correct, but is not the whole story. Now, we apply the same logic to the case of two antennas driven by out-of-phase currents. Because the antennas are so closely spaced, the radiated electric and magnetic fields cancel completely, and there is consequently no radiated electromagnetic power. This is indeed true. Yet, if we try to find the power by using Eq. (1) applied to each antenna of the two, we are forced to the conclusion that we are "pumping" I amperes through a 73 Ω resistance, and therefore the power to each of the two antennas should be nonzero. Currents are indeed flowing through both generators, so the absence of radiated power seems to contradict Eq. (1). This author has witnessed some pseudoexplanations of this fact, which attempt to assume that some increasing electromagnetic energy is being stored in the small space between the two antennas, or that the two antennas can never physically be placed close enough together to completely cancel their fields (even through the corresponding assumption that they can be close enough to exactly double their fields in the earlier case is accepted).

The answer is that the "sacred cow" in the expression is the invariable use of 73 Ω as the antenna impedance. The "impedance" of an electromagnetic antenna is a derived quantity, found correctly only after computing the true external fields produced by the antenna and all other conductors in the vicinity. Thus, the true impedance of one antenna, when the other is driven 180° out of phase with it, is actually 0 Ω. Similarly, when the other antenna is driven in phase, the impedance of each member of the first antenna set is then double its "isolated" value, or 146 Ω. That is another, better, way of explaining why the power into the composite in the in-phase case is quadruple that of a single antenna.

APPLICATION TO DERIVING ANTENNA IMPEDANCE

An interesting incidental result of this change in impedance due to another nearby in-phase antenna is that the impedance of the so-called "folded" dipole antenna, used as the main receiving element of most television antennas, is 292 (= 4·73) Ω (see Fig. 2). The 292 Ω impedance of the folded dipole is the reason for the use of so-called "300 Ω" two-wire flat cable. This 300 Ω characteristic surge impedance is a close match to 292 Ω, while a single (73 Ω) center fed dipole is usually fed by means of a coaxial cable having a 75 Ω characteristic surge impedance. The in-phase current in the second leg of the folded dipole antenna is produced by electromagnetic induction from the primary driven leg, and not by a separate generator. The connection of the two legs at their ends does not affect the field pattern significantly because the ends of a half wave dipole have nodes of current, i.e., zero current.[3]

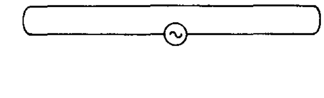
ACOUSTIC EXAMPLE

In addition to the electromagnetic radiation from an antenna, a similar situation exists with an acoustic transducer, such as a loudspeaker. In many cases, arrays of small loudspeakers are mounted together on a planar or spherical support. While the electrical impedance of an individual isolated loudspeaker may be typically 8 Ω, the effect of mounting a large number of loudspeakers together, when all are phased alike, is to change the electrical impedance of each participating loudspeaker. The approximate value of loudspeaker electrical impedance in a planar array will be simply the characteristic acoustic wave impedance of air, scaled by a conversion factor describing the ratio of mechanical force to electric current in the driving coil of the loudspeaker. This depends upon the magnetic strength of the driver permanent magnet and the number of active turns in the loudspeaker driving coil. It is independent of the size or shape of the loudspeaker.

Conversely, many readers may be familiar with the demonstration in which a loudspeaker is suspended inside a bell jar and operated to produce a sound while the air is exhausted from the jar by a vacuum pump. The major purpose of this demonstration is to illustrate that the listeners can no longer hear the sound when the air is removed. However, students often ask about the power being delivered electrically to the loudspeaker by the wiring. Where does it "go"? Because the characteristic acoustic wave impedance of air approaches zero as the density approaches zero, the reflected electrical impedance of the loudspeaker will also approach zero in this experiment. To be more exact, the electrical impedance of the loudspeaker will become purely reactive (neglecting the electrical resistance of the wire in its coil). Energy is stored and returned during each cycle of alternating current due to the mechanical elasticity of the deformations of the loudspeaker cone.

Of course, the above example of an acoustic transducer operating in a vacuum is not a case of the use of superposition of sources. However, if one uses two closely spaced acoustic transducers, one may drive them out of phase to produce almost total cancellation of their acoustic radiation. This is precisely analogous to the case of electromagnetic antennas described in Fig. 1. We define the expression "close together," for both the electromagnetic and the acoustic cases, as a spacing between corresponding portions of the two radiators which is much smaller than a quarter wavelength.

Fig. 2. Folded dipole antenna is produced from the double antenna of Fig. 1 by connecting the ends of both and replacing the second current source by a short circuit. Current is produced in the second leg by induction rather than by a second source.



Various attempts have been made to apply this method to the production of a true "sound canceling" device.[4] However, most such devices are not able to place an out-of-phase transducer close to the source of sound to be canceled. Even though these systems are reasonably successful at picking up the ambient sound or noise with a strategically placed microphone and producing an out-of-phase acoustic output with a loudspeaker, the cancelation effect is limited to a small volume of space. Further away from the canceling loudspeaker there may be isolated small regions where the sound in specific frequency ranges is cancelled. However, by moving one's ear slightly the sound can again be heard. The overall far field pattern of sound cancelation and reenforcement is, of course, the classical Fraunhofer diffraction pattern due to two sources—the original source and the cancelation loudspeaker. In addition, we should note that the spatial pattern will be different for each frequency component of the sound, because the spacing of the maxima and minima of the diffraction pattern will be more widely spaced at low frequencies corresponding to longer wavelengths.

After this lengthy preliminary discussion, we finally address the question raised by Clarke's story. If an active device such as a "sound canceler" described above is used in a limited space where it achieves almost perfect cancelation, what of the power produced by the original sound source? And does the "sound canceler" itself produce any power? As one can infer from our preliminary discussion, the case where the two sources are extremely close, and produce complete cancelation, leads to no power radiated by either source. As the two sources are physically separated, the wave impedance seen by either source (for a single frequency test signal) would vary between maxima and minima as the two sources were separated by increasing half wave distances, but the individual maxima and minima would become less extreme until the wave impedance approached the nominal value, independent of position, when the sources are very far separated. The minima in wave impedance correspond to separations of integral half wavelengths, where there is a somewhat greater volume of far-field cancelation than reenforcement, while the maxima correspond to odd numbers of quarter wavelength separations, for which there is a somewhat greater volume of far field reenforcement than cancelation. At very large separations of the sources, the far field diffraction pattern has approximately equal volumes of cancellation and reenforcement, leading to almost no net change in the radiated power from each source as compared to a single isolated source.

Consider again the case of very close spacing, where complete cancelation is achieved and neither source radiates any power. Note that one must provide a "power" amplifier with the capability of driving enough electric current through the moving coil of the cancelation loudspeaker to produce the desired motion of the loudspeaker core. Thus we must provide an amplifier with the *capability* of producing 100 W effective acoustic output power, or at least the capability of providing the corresponding undistorted output current, in order to cancel 100 W of acoustic ambient power. This even though the amplifier will deliver no power during its actual use for cancelation.

As a practical matter, the amplifier must have the capability to drive a controled current through a 0 Ω circuit. In electrical jargon, the amplifier must have a high output impedance. It "looks like" an ideal current source. There is no question of attempting to match the output impedance of the amplifier to the low impedance of the load, because no power is being transferred.

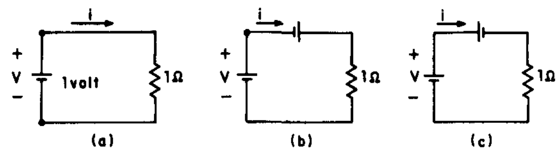


Fig. 3. "Impedance" seen by the labeled battery is 1 Ω in (a), apparently ½ Ω in (b), and apparently infinite in (c).

Prior to the final draft of this paper, a study of coupled motion of piano strings was published by Weinreich.[5] He showed that the oscillations from two adjacent pianoforte strings which vibrate in a cooperative mode of in-phase oscillation die out much more rapidly than a single isolated string. This is due to the fact that they loose their energy at four times the rate of a single string alone. This is another acoustic analog of the two antennas of Fig. 1.

CIRCUIT EXAMPLES

Part of the problem in explaining the wave examples above is that students may not understand wave impedance, and yet still want an explanation of the phenemena involved. Similar conclusions can be drawn from analogous electrical circuits, without use of waves. In engineering terminology, the following examples are "lumped" rather than "distributed."

An analogous electrical circuit to Fig. 1 is shown in Fig. 3, which shows a simple circuit with a 1 Ω resistor and some 1 V batteries. With only one battery connected, the impedance of the resistor, computed as V/I, is 1 Ω as expected. Two batteries connected in series, aiding, produce a total of 2 A current. If we wish to find the "effective impedance" *seen by only one of the batteries*, the ratio of V/I is ½ Ω! The alert student will object to computing the "effective impedance" of a circuit which is not passive and contains internal power sources, i.e., the other battery! Such an objection is completely correct. It is equally wrong to speak of the intrinsic impedance of an antenna or a loudspeaker measured while other nearby interfering antennas or loudspeakers are operating, and for the very same reason. Note, however, that an antenna with nearby *passive* elements, such as the folded dipole, has a legitimately different true passive impedance than a single dipole, because it has a different geometrical configuration than a single dipole. The conceptual determination of impedance in that case, by treating the induced current in the second leg of the dipole as the effect of a hypothetical second source, is a legitimate method if the second source is considered to be proportional to the actual physical driving current in the primary leg, and not an independent noncontroled source of current.

SOURCES OF INVARIABLE POWER

Previous errors in superposition of fields or currents occur when one makes the error of attempting to treat a source as though it produced an invariable power, rather than a specified force or velocity (in the mechanical case) or current or voltage (in the electrical case). For purposes of clarification to the student, one may postulate a hypothetical device which always delivers a fixed amount of power, so it can be compared with more realistic models of devices which produce fixed current or voltage. Such an electrical device would have the hypothetical current-voltage relationship

$$vi = -P, \tag{2}$$

where P is the constant current-voltage product (power) which the source maintains. The negative sign on P in the equation implies that the device delivers, rather than absorbs, power.

If such a hypothetical device were connected to a resistor, it would always deliver the same power P, by producing a voltage and current magnitude of value

$$v = sqrt(PR), \tag{3}$$

$$i = sqrt(P/R), \tag{4}$$

so that the product of v and i would invariably be P. There is obviously a mathematical inconsistency for the cases of a short circuit ($0\ \Omega$) or an open circuit (infinite ohms). For example, if the hypothetical power source were connected to a short circuit, the power source definition implies that the current-voltage product be nonzero, while the short circuit demands that the voltage, and thus the current-voltage product, be zero. Contrast this with the cases of connecting an ideal current source to a short circuit and an ideal voltage source to an open circuit, which are both consistent realizable circuits, and the two cases of connecting a current source to an open circuit and connecting a voltage source to a short circuit, which are both inconsistent and not physically possible. In the latter two cases of the previous sentence, as well as any attempted real physical implementation of the hypothetical power source, the real physical device will always change to some failure mode to solve the physical problem! A fuse or circuit breaker will open the circuit, a transistor will melt, or some other effect not included in the simplified mathematical model of the device will come into play to resolve the difference between reality and the inconsistent theoretical description.

TEXTBOOK DISCUSSIONS

The subject of conservation of energy in the destructive interference of waves is treated explicitly in only two out of over 50 textbooks which the author has examined.[6,7] Both show that in the case of two slit interference, the far field average intensity of illumination is the same as the sum of the intensity due to each slit acting alone. No text has been found which explains the case of total far field cancelation. Such an effect could be demonstrated for two slits if they are first very closely spaced compared to the wavelength of the illumination, and also if a half wave plate is placed over one slit to put its transmitted radiation 180° out of phase with the illumination transmitted by the other. This demonstration cannot be constructed with practical dimensions for visible light, but a demonstration of practical size can be made for radio wavelengths.

In the structure just described, the radiation emerging from one slit spreads in all directions, including a path backwards through the other slit. Because of its 180° phase, the back ray is identical to the reflection from a conductive mirror. The two slit structure with half wave plate thus reflects all of the incident energy back to the source (ideally), despite the fact that the slits are perfectly transparent! Because this result is not obvious and the analysis is not readily available to the student, the misconceptions described in this paper have attained some currency.

[1]I. Asimov, Sci. Digest **53** (Oct. 1972).

[2]A. C. Clarke, in *Tales From the White Hart* (Harcourt Brace and World, NY, 1970; in paperback by Ballentine Books, NY, 1957).

[3]J. D. Kraus, *Antennas*, pp. 143ff and 262ff, (McGraw-Hill, New York, 1950) or other standard texts on the subject.

[4]H. F. Olson and E. G. May, J. Acoust. Soc. Am. **25**, 1130–1136 (1959); H. F. Olson, *Acoustical Engineering* (Van Nostrand, Princeton, NJ, 1957).

[5]G. Weinreich, Sci. Am. **240**, 118 (Jan. 79).

[6]F. A. Jenkins and H. E. White, *Fundamentals of Physical Optics*, 4th ed. (McGraw-Hill, NY, 1976), p. 265.

[7]J. Strong, *Concepts of Classical Optics* (Freeman, San Francisco, 1958), p. 163.