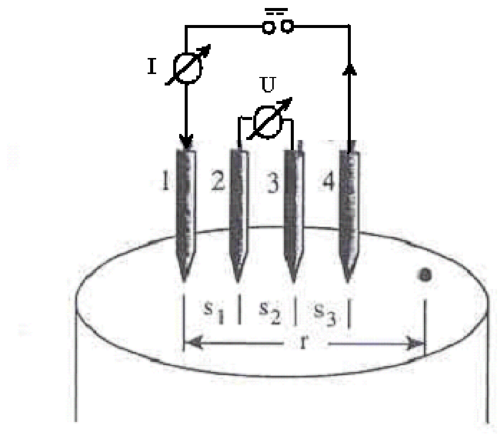
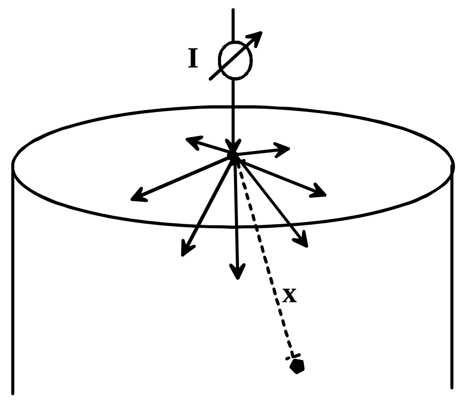
Prof. Dr. H. Föll
Chair for general materials science

Exercises "Advanced materials B"

Exercise 2: The four-point probe





(left): Collinear four point probe.

(right): Current *I* enters a semi-infinite sample through one probe.

To measure the resistivity ρ of wafers, thin films etc., the four-point-probe method is mostly used. The left part of the figure shows a collinear four-point probe arrangement. A current is flowing from probe 1 to probe 4. Between probe 2 and 3 the voltage drop is measured. The current and voltage are measured with different probe pairs, because if one would measure the voltage between probe 1 and 4 as well, one would measure a lot higher resistance than the simple material resistance.

a) Write down qualitatively the resistance chain between probe 1 and 4.

We first calculate the current and electrical field distribution for one probe (probe 1), from which the current I is flowing into an idealized ***semi-inifinite*** sample (cf. right part of the figure) of constant resistivity ρ:

b) Show that the current density at a distance x_1 from probe 1 is given by $j(x_1) = \frac{I}{2\pi x_1{}^2}$.

c) Calculate the electrical field strength distribution $E(x_1)$ by using Ohm's law (Solution: $E(x_1) = \frac{\rho I}{2\pi x_1^2}$).

d) The reference potential at infinite distance from the probe is U_∞ := 0 V. Derive that the potential at distance x_1 from the probe is $U(x_1) = -\frac{\rho I}{2\pi x_1}$.

Probe 4can be treated similar to probe 1, the only difference is that I is negative because the current is leaving the sample there.

e) Explain shortly why you are allowed to sum up the potentials "originating" from probe 1 and 4!

f) Determine the potential at a distance x_4 from probe 4 where the current I leaves the sample!

g) Show that the potential difference between probe 2 and 3 is $\Delta U = -\frac{\rho I}{2\pi s}$, if they are equally spaced (spacing s).

h) What is the resistivity ρ of the sample?

This example can be generalized to account for arbitrarily shaped samples: The resistivity can then be formulated as $\rho = F\frac{V}{I}$, where F is a geometrical correction factor (in the case of a semi-infinite sample: $F = 2\pi s$). For a thin sample, a thickness $t \leq \frac{s}{2}$, a measurement in the middle of the sample, and a sample's lateral extension of at least 4s, $F = \frac{t\pi}{\ln 2}$ results, i.e. the measurement is independent of the probe spacing s!

i) For a commercially available semiconductor wafer with t = 525 µm, a voltage U = 12 mV is measured for a current I = 10 mA. Calculate the resistivity ρ of the wafer!