

Solution to Exercise 3.2-3

Illustration

How large will be the distance d between the (center of gravity) of the positive and negative charges for reasonable field strengths and atomic numbers, e.g. the combinations of

- 1 kV/cm
- 100 kV/cm
- 10 MV/cm
- , the last one being about the ultimate limit for the best dielectrics there are,
- $z = 1$ (H, Hydrogen)
- $z = 50$ (Sn (= tin), ...)
- $z = 100$ (?)

From the backbone we have a relation for d as a function of z , m the radius R of the atom, and the field strength E :

$$d_E = \frac{4 \pi \epsilon_0 \cdot R^3 \cdot E}{ze}$$

We need to look up some number for the radius of the three atoms given (try this link), then the calculation is straight forward - let's make a table:

Atom	R	$d(1 \text{ kV/cm})$	$d(100 \text{ kV/cm})$	$d(10 \text{ MV/cm})$
$z = 1$				
$z = 50$				
$z = 100$				

Compared to the radius of the atoms, the separation distance is tiny. No wonder, electronic polarization is a small effect *with spherical atoms!*

Calculate the "spring constant" and from that the resonance frequency of the "electron cloud" (assume the nucleus to be fixed in space).

If you don't know off-hand the resonance frequency of a simple harmonic oscillator - that's fine. If you don't know exactly what that is, and where you can look it up - you are in deep trouble.

Anyway, in [this link](#) you get all you need. In particular the resonance (circle) frequency ω_0 of a harmonic oscillator with the mass m and the spring constant k_S is given by

$$\omega_0 = \left(\frac{k_S}{m} \right)^{1/2}$$

How large are the spring constants? That is question already answered in the backbone, so we import the equation

$$k_S = \left(\frac{(ze)^2}{4 \pi \epsilon_0 \cdot R^3} \right)$$

Again, let's make a table for the answers:

Atom	Spring constant	ω_0
z = 1		
z = 50		
z = 100		