

### 3.2.5 Summary and Generalization

For all three cases of polarization mechanisms, we had a *linear* relationship between the electrical field and the dipole moment (for fields that are not excessively large):

#### Electronic polarization

$$\mu_{EP} = 4\pi \cdot \epsilon_0 \cdot R^3 \cdot E$$

#### Ionic polarization

$$\mu_{IP} = \frac{q^2}{k_{IP}} \cdot E$$

#### Orientation polarization

$$\mu_{OP} = \frac{\mu^2}{3kT} \cdot E$$

It seems on a first glance that we have justified the "law"  $P = \chi \cdot E$ .

- However, that is not quite true at this point. In the "law" given by equation above,  $E$  refers to the *external* field, i.e. to the field that would be present in our capacitor *without* a material inside.
- We have  $E_{ex} = U/d$  for our plate capacitor held at a voltage  $U$  and a spacing between the plates of  $d$ .
- On the other hand, the induced dipole moment that we calculated, always referred to the *field at the place of the dipole*, i.e. the *local* field  $E_{loc}$ . And if you think about it, you should at least feel a bit uneasy in assuming that the two fields are identical. We will see about this in the next paragraph.

Here we can only define a factor that relates  $\mu$  and  $E_{loc}$ ; it is called the **polarizability**  $\alpha$ . It is rarely used with a number attached, but if you run across it, be careful if  $\epsilon_0$  is included or not; in other words what kind of [unit system](#) is used.

- We now can reformulate the three equations on top of this paragraph into one equation

$$\underline{\mu} = \alpha \cdot E_{loc}$$

- The **polarizability**  $\alpha$  is a material parameter which depends on the polarization mechanism: For our three paradigmatic cases they are given by

$$\begin{aligned} \alpha_{EP} &= 4\pi \cdot \epsilon_0 \cdot R^3 \\ \alpha_{IP} &= \frac{q^2}{k_{IP}} \\ \alpha_{OP} &= \frac{\mu^2}{3kT} \end{aligned}$$

- This does not add anything new but emphasizes the proportionality to  $E$ .

So we *almost* answered our [first basic question](#) about dielectrics - but for a full answer we need a relation between the *local* field and the *external* field. This, unfortunately, is *not a particularly easy problem*

- One reason for this is: Whenever we talk about electrical fields, we always have a certain scale in mind - without necessarily being aware of this. Consider: In a metal, as we learn from electrostatics, there is *no field at all*, but that is *only true* if we do not look too closely. If we look on an *atomic scale*, there are tremendous fields between the nucleus and the electrons. At a somewhat larger scale, however, they disappear or perfectly balance each other (e.g. in ionic crystals) to give no field on somewhat larger dimensions.

- The scale we need here, however, is the *atomic scale*. In the electronic polarization mechanism, we actually "looked" *inside* the atom - so we shouldn't just stay on a "rough" scale and neglect the fine details.

Nevertheless, that is what we are going to do in the next paragraph: *Neglect the details*. The approach may not be beyond reproach, but it works and gives simple relations.

## Questionnaire

Multiple Choice questions to all of 3.2